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AN OPTIMIZATION STUDY OF BLAST SHELTER DEPLOYMENT

David L. Mitchell

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Appendices A-G

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AN OPTIMIZATION STUDY OF BLAST SHELTER DEPLOYMENT

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VOLUME II: APPENDICES A-G

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APPENDIX A

ADDITIONAL LAGRANGE MULTIPLIER THEORY

David L. Mitchell

APPENDIX A

ADDITIONAL LAGRANGE MULTIPLIER THEORY

A. GENERAL

The presentation here assumes familiarity with Section IIID of the basic report. The notation is the same, with K representing attack level, H the payoff, etc. First we present two theorems regarding single-sided maximization:*

Lambda Theorem: Suppose λ_1 and λ_2 produce solutions X_1^* and X_2^* and suppose $K(X_1^*) \geq K(X_2^*)$.

Then $\lambda_2 \geq \dfrac{H(X_1^*) - H(X_2^*)}{K(X_1^*) - K(X_2^*)} \geq \lambda_1$.

In the limit for a continuous case this becomes $\dfrac{\partial H^*}{\partial K} = \lambda$. This theorem indicates that lower lambdas produce higher resource consumption, and hence provides the information needed for consumption-closing by iteration. Also contained in the theorem is the following important bounding corollary:

> Corollary: If λ_1 produces a solution X_1^*, then for any other resource level $K(X)$, $H(X) \leq H(X_1^*) + \lambda_1 \left[K(X) - K(X_1^*)\right]$.
>
> This corollary provides an upper bound for payoffs for resource levels other than the one found in the optimization.

Epsilon Theorem: Suppose $\bar{X}$ comes within ϵ of maximizing the Lagrangian, i.e., for all X,

$$H(\bar{X}) - \lambda K(\bar{X}) > H(X) - \lambda K(X) - \epsilon .$$

*Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources," Hugh Everett, III, Operations Research, Vol.11,No.3.

Then $\bar{X}$ is a solution of the constrained problem with constraint $A = K(\bar{X})$ that is itself within ϵ of the maximum for that constraint.

This theorem is useful when an iterative scheme is used for maximization or when an approximation is used for a true function.*

With the main theorem and the above theorems, we have the material to handle one-sided maximization. The main theorem gives the procedure for finding maxima. The lambda theorem gives the strategy for iterative resource level matching, and the corollary and the epsilon theorem provide error bounds if resource levels are not exactly matched or the maximum Lagrangian is not found exactly. It should be noted that all solutions found by this method are rigorous, although it is possible that solutions do not exist (and hence will not be found). This is discussed fully in Everett's paper.

There is a theorem for the two-sided Lagrange procedure related to the epsilon theorem. This theorem applies to the case in which an attack is computed by an approximation to the real damage function. If we know the error in the approximation, how much better than indicated can the defender expect to do? For the double Lagrange we can only assess this error in the Lagrangian itself.

Approximation Theorem: Suppose the payoff function $H(X,Y)$ is approximated with $\tilde{H}(X,Y)$ such that

$$\tilde{H}(X,Y \geq H(X,Y) - \epsilon \quad \text{and}$$

$$\tilde{H}(X,Y) \leq H(X,Y).$$

*See Section IVD of the main report for an example of the use of this theorem.

Suppose that $\tilde{H}(X,Y)$ is minimaxed to find a defense Y^* and an optimal attack X^*. Then

$$1)\ \max_X \left[H(X,Y) - \lambda K(X) + \mu C(Y)\right] \geq H(X^*,Y^*) \quad \lambda K(X^*) + \mu C(Y^*) - \epsilon$$

That is, the true minimax Lagrangian is less than ϵ below the true Lagrange evaluation of the optimal defense and attack.

$$2)\ \max_X \left[H(X,Y) - \lambda K(X) + \mu C(Y)\right] \geq \tilde{H}(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*)$$

That is, the true minimax Lagrangian is greater than the approximate solution.

Proof: Since the Lagrangian is minimaxed,

$$\tilde{H}(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*) \leq \max_X \left[\tilde{H}(X,Y) - \lambda K(X) + \mu C(Y)\right]$$

Using on the right the second supposition we get the first result:

$$\tilde{H}(X^*, Y^*) - \lambda K(X^*) + \mu C(Y^*) \leq \max_X \left[H(X,Y) - \lambda K(X) + \mu C(Y)\right]$$

Using on the left the first supposition we obtain

$$H(X^*,Y^*) - \lambda K(X^*) + \mu(Y^*) - \epsilon \leq \max_X \left[H(X,Y) - \lambda K(X) + \mu C(Y)\right]$$

This result tells us how errors will affect the Lagrangian--first, the approximate Lagrangian solution is less than the true Lagrangian if the approximation underestimates kill. More importantly, we are guaranteed that if we used the true payoff function we could only improve the defense effectiveness by ϵ; i.e., the defense Y^* which we find is within ϵ of the best the defense can do.

For the two-sided Lagrange procedure there is no theorem corresponding to the main theorem in the previous section. Results may be erroneous, in that solutions may be found which are not the best possible choices for the defense. However, these spurious solutions are not troublesome for problems

with large numbers of cells, for the possible errors become very small. All defenses found in the study were calculated using only the theory presented up to this point. Nevertheless, the possibility of error should be considered, and spurious solutions will be discussed now.

It is very instructive to attempt to prove an extension of the main theorem for the two-sided case. Suppose we find X^* and Y^* to minimax the Lagrangian, i.e.,

$$H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*) \leq \underset{X}{\text{Max}} \left[H(X,Y) - \lambda K(X) + \mu C(Y) \right]$$

On the left we have found $H(X^*,Y^*)$ for which

$$H(X^*,Y^*) \geq H(X,Y^*) \text{ for all } X,\ K(X) \leq K(X^*).$$

It would seem we could apply the one-sided theorem to the right. We can, and we find $\hat{X}$ such that $H(\hat{X},Y) \geq H(X,Y)$ for all X, $K(X) \leq K(\hat{X})$. Note, however, that since $Y \neq Y^*$, the two one-sided maximizations were effectively performed with different payoff functions, $H(X,Y^*)$ and $H(X,Y)$. The two attack levels $K(X^*)$ and $K(\hat{X})$ result from those maximizations, and $K(X^*) \neq K(\hat{X})$! Thus, we cannot cancel the terms and we cannot even determine which attack level is larger. Thus the attempt breaks down.

As an example suppose we have two equal cost defenses Y_1 and Y_2 with payoff functions shown for each in Figure A-1 . Maximizing the attacker's Lagrangian using the lambda value λ_1 for defense Y_1 leads us to A with Lagrangian value a, for Y_2 to B with value b. Since the costs are equal, it can be seen that we minimize the maximum Lagrangian by selecting A as the best point and Y_1 as our defense. But observe that at the attack

level at A, Y_2 is a better defense. Now also notice that if lambda is decreased to λ_2, then point B will be chosen over point A and defense Y_2 will be chosen, which for the attack level at B is again the wrong defense. If lambda

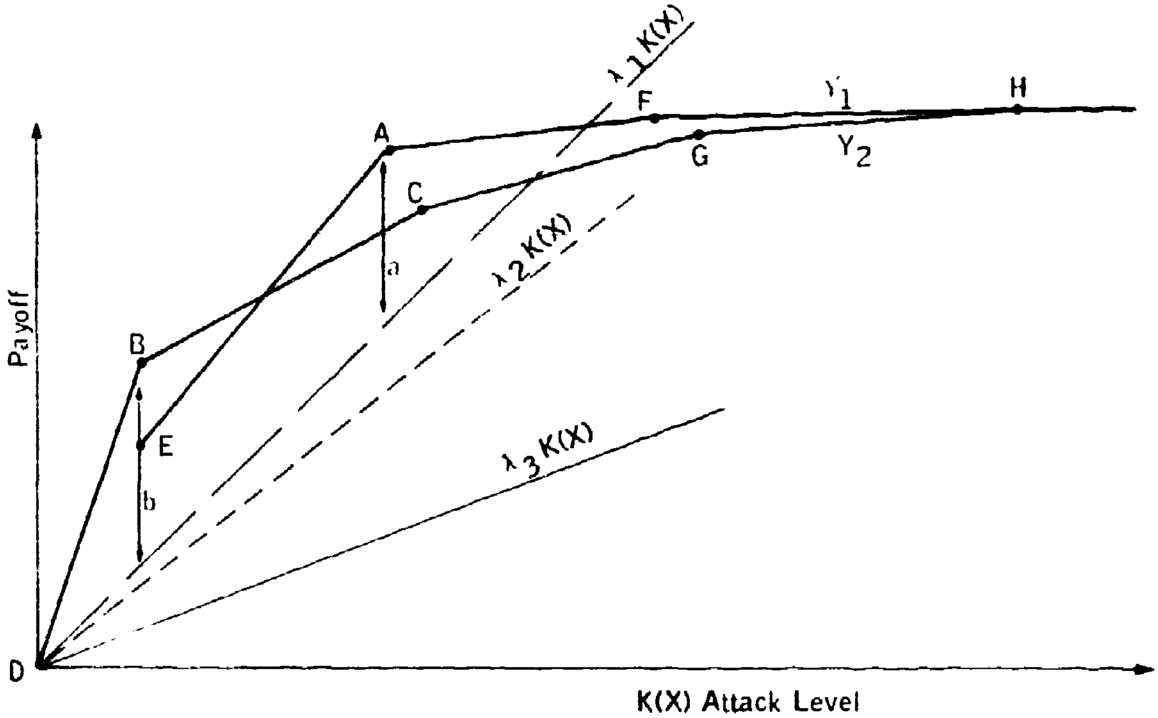


Figure A-1. Example of Spurious Solutions

is decreased still more to λ_3 the contest will be between points C and A, with defense Y_2 being chosen, which is correct for attack level at C. Thus we see that the double Lagrange method may find both valid solutions (e.g., point C) and spurious solutions.

Now look again at Figure A-1. If we imagine beginning with a high lambda and reducing it, we will find that we will pick up solutions in the following order: D, E, A, B, C, G, H. There are points which are never found, such as F,

and the double-Lagrange procedure will screen out many such points. If we connect the points for $Y_1(D, E, A)$ and Y_2 (D, C, G, H), or if we simply connect all points in the order found, we find we can locate the optimal curves. Consideration of this capability to find the optimal curve will lead to the theorem which follows. First, however, let us define things a bit more precisely.

We are interested in determining which solutions generated by the double Lagrange method are spurious.* This is, of course, equivalent to finding the optimal curve, for any non-spurious solutions lie on the optimal curve. The optimal curve for some cost is defined to be

$$\theta(A,B) = \underset{Y,C(Y)\leq B}{\text{Min}} \quad \underset{X,K(X)\leq A}{\text{Max}} \quad H(X,Y).$$

Let us also make the assumptions about the attacker's payoff explicit. We consider the maximum payoff for some attack level, $G(A,Y) = \underset{X,K(X)\leq A}{\text{Max}} H(X,Y)$. Concerning this maximum payoff, we assume

1) $G(A,Y)$ is a continuous function of A.
2) $\frac{d}{dA}$ $G(A,Y) \geq 0$ (but possibly discontinuous)
3) $\frac{d}{dA}$ $G(A,Y)$ is a monotone nonincreasing function of A.**

---

* One point should be made here: the spurious solutions are actual defenses and the payoff for each defense is its true and proper payoff; the spurious nature is only that there is a better solution. Thus the separate and independent evaluation used in the computer program does not change the nature of the solution. It may be, however, that some "spurious" solution is the best solution for some set of defense objectives.

**"Lagrange Multipliers and the Optimal Allocation of Defense Resources", George E. Pugh, Operations Research, Vol. 12, No. 4, has a thorough discussion of how to handle gaps, i.e., regions where these assumptions are not valid.

Many times it will also be true that $\Theta(A,B)$ is a concave upward function of B, i.e., $G(A,Y) \geq \Theta(A,B) + \mu B - C(Y)$

Existence Theorem: Any point on an optimal curve which is concave upward with respect to cost can be located with the Lagrangian procedure;* i.e., there exist λ and μ such that

$$H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*) \leq \underset{X}{\text{Max}} \left[H(X,Y) - \lambda K(X) + \mu C(Y) \right].$$

Proof: Suppose to the contrary that for every λ and μ there is a Y^0 such that

$$\underset{X}{\text{Max}} \left[H(X,Y^0) - \lambda K(X) + \mu C(Y^0) \right] < H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*)$$

If the inequality is true with maximization over all X, it is true over the subset for which $K(X) = K(X^*)$, and $K(X)$ is achievable because of the convexity with respect to X. Then

$$\underset{X,K(X)=K(X^*)}{\text{Max}} \quad H(X,Y^0) + \mu C(Y^0) < H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*)$$

Or, $\underset{X,K(X)=K(X^*)}{\text{Max}} H(X,Y^0) + \mu C(Y^0) < H(X^*,Y^*) + \mu C(Y^*)$

which contradicts the concavity assumption.

The usefulness of this theorem is that it guarantees that if we examine all lambdas and mus we have found <u>all</u> solutions of interest. If the computation procedure has an efficient cost closing mechanism, then all solutions can be found by sweeping through all lambdas. If distinct defenses exist, then spurious solutions will abound.

---

* Without the assumption of concavity in cost and convexity in attack level the theorem could be proved for a single cost with no restriction on concavity. However, these difficulties have to do with gaps, and the theorem is directed toward assuring that even with spurious solutions, the optimal curve can still be found.

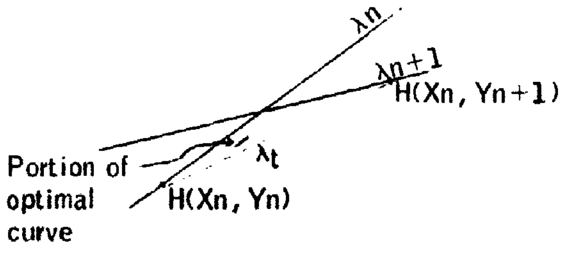
Suppose we find Lagrangian solutions by sweeping lambdas for a fixed cost; we may distinguish two types of "adjacent" solutions as we sweep lambdas:

1. Both adjacent solutions are the same defense. This is true if and only if the resultant attack level shift is with the prevailing lambda trend. About these points we can say nothing--they are not obviously spurious.
2. The adjacent solutions are different defenses. The resultant attack level shift may be either direction, but is probably opposite to the lambda trend. These points are always spurious, but it should be remembered that they will give information to finding the optimal curve.

The following theorem enables us to find the optimal curve by a simple graphical method.

<u>Construction Theorem</u>: The optimal curve is the lower envelope of lines connecting in order all solutions found in the lambda sweep.

Proof: The existence theorem tells us that every point on the optimal curve can be found with some lambda (here we consider the fixed cost case). Thus all we need to show is that we do not draw a line below the optimal curve. Suppose we have two solutions H(Xn, Yn) and H(Xn+1, Yn+1) for λn and $\lambda n+1$



with no other distinct solutions for intermediate lambdas. Supose there is a portion of the optimal curve above the line with slope λ_t which joins the two points. The optimal curve cannot cross either line λn or line $\lambda n+1$ for by convexity, defenses Yn and Yn+1 are always below those lines, and that would contradict the optimality conditions. Thus the optimal curve must cross the λ_t line segment twice and hence the optimal curve achieves a maximum with respect to λ_t; therefore there exists a distinct solution to the Lagrangian formed with λ_t. But $\lambda n+1 < \lambda_t < \lambda n$, which contradicts our assumption that there were no intermediate solutions.

B. BOUNDS FOR THE OPTIMAL CURVE

The continuous lambda sweep we have been discussing will not usually be feasible. The question can then be raised as to how small a step is allowable to accomplish the same purpose. However, for any step, no matter how small, it is possible to construct a point on the optimal curve which will not be located. Certainly, though, we achieve an approximate location for the optimal curve, and the verification procedure addresses itself to the task of finding rigorous bounds for the optimal curve, with no other assumptions than the ones on convexity. More knowledge of the functions involved could probably provide better bounds.

We now give the theorem from Pugh's paper which provides an upper bound on the optimal curve.

Upper Bound Theorem: If X* and Y* are solutions to the Lagrangian, for all A,

$$\Theta\left[A, C(Y^*)\right] \le H(X^*, Y^*) + \lambda\left[A - K(X^*)\right] .$$

The upper bound theorem is for only a single cost, $C(Y^*)$. If it is reasonable to assume concavity (and if it is not, solutions will not exist), then interpolation can be used to determine the upper bound for any cost between solutions for two different costs.

Before we prove the lower bound theorem we will consider a lemma, which is useful in its own right for understanding spurious solutions.

Lemma: Suppose that X^* and Y^* are solutions of the Lagrangian for λ and μ. Then for every defense Y there exists some X' such that

$$H(X',Y) - \lambda K(X') + \mu C(Y) \ge H(X^*, Y^*) - \lambda K(X^*) + \mu C(Y^*)$$

i.e., for every defense Y there is at least one point which lies above the plane $H(X^*,Y^*) + \lambda\left[A - K(X^*)\right] - \mu\left[B - C(Y^*)\right]$, which we denote P(A, B).

Proof: By the minimization of the Lagrangian over Y,

$$H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*) \le \underset{X}{\mathrm{Max}} \left[H(X,Y) - \lambda K(X) + \mu C(Y)\right]$$

But since the right hand side must achieve its maximum, there exists some X' such that

$$H(X',Y) - \lambda K(X') + \mu C(Y) \ge H(X^*,Y^*) - \lambda K(X^*) + \mu C(Y^*)$$

Let us now see how to use the Lemma to bound possible solutions. Figure A-2 shows the lemma surfaces in the plane of cost

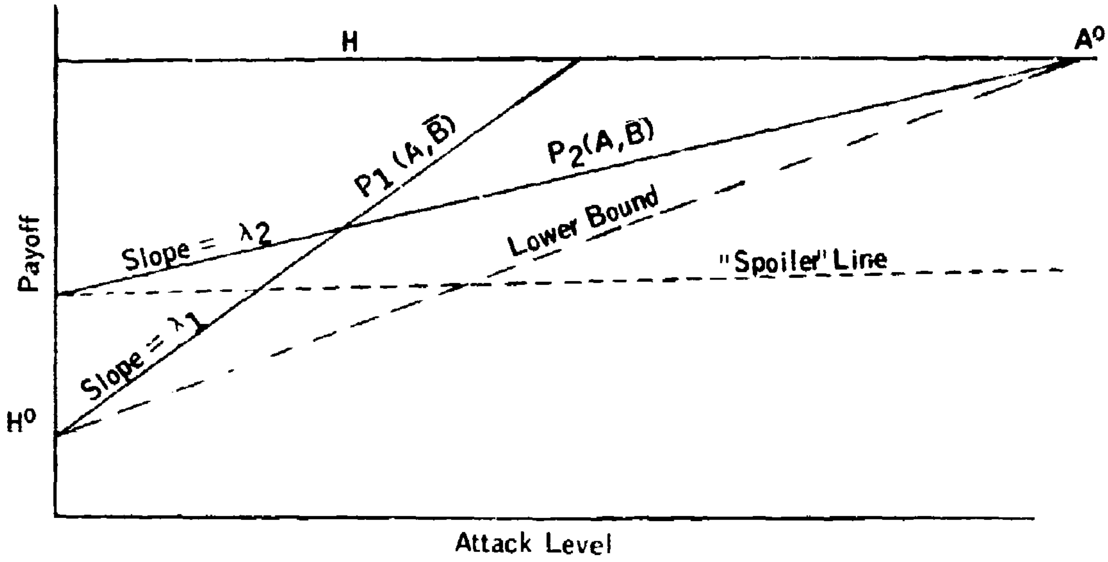


Figure A-2. The Lower Bound

$\bar{B}$ for two solutions to the Lagrangian for λ_1 and λ_2 (the solution points do not appear in the figure since they need not be of cost $\bar{B}$). The lemma states that any defense not found must lie above P_1 at some point and above P_2 at some other point. We now ask where this requirement together with convexity allows this other defense to wander. We first consider only parts of the optimal curve with slope between λ_1 and λ_2. Some consideration will show that the worst possible condition for convex curves will require that $\bar{G}(A,\bar{Y})$ go straight from the origin to point H^o, then to point A^o, as shown in the figure. That line then is the lower bound for all parts of the optimal curve with slope between λ_1 and λ_2. The "spoiler" line shows why the requirement that the curve lies somewhere above P_1 and P_2 does not give that bound for portions of the curve with any slope whatever, in that it clearly lies below the lower bound line.

The lower bound theorem makes this more precise:

<u>Lower Bound Theorem</u> Suppose that X_1, Y_1 and X_2, Y_2 are solutions of the Lagrangian for λ_1 and λ_2 (with $\lambda_1 > \lambda_2$) and μ_1, μ_2. Then any portion of $\Theta(A,B)$ with slope λ, $\lambda_2 \le \lambda \le \lambda_1$ lies on or above the surface determined by

$$\frac{H}{A^0} \quad A + H^0\left(1 - \frac{A}{A^0}\right)$$

for every cost B. In the above

$$H^0 = \text{Min}\left\{ H(X_1,Y_1) - \lambda_1 A_1 - \mu_1 (B-B_1), \right.$$
$$\left. H(X_2,Y_2) - \lambda_2 A_2 - \mu_2 (B-B_2) \right\}$$

$$A^0 = \text{Max}\left\{ A_1 + \frac{1}{\lambda_1} \left[H - H(X_1, Y_1) + \mu(B-B_1) \right] \right.$$
$$\left. A_2 + \frac{1}{\lambda_2} \left[H - H(X_2,Y_2) + \mu(B-B_2) \right] \right\}$$

Proof: Consider $\Theta(A,B)$ some point of $\Theta(A,B)$ with slope $\bar{\lambda}_1$, $\lambda_2 \le \bar{\lambda} \le \lambda_1$ and let $\bar{Y}$ be the defense at this point. From the lemma,

$H(X,\bar{Y}) \ge P_1(A,\bar{B})$ for some X (P_1 is the plane associated with λ_1), and hence

$G(A,\bar{Y}) \ge P_1(A,\bar{B})$ for some A. Similarly

$G(A,\bar{Y}) \ge P_2(A,\bar{B})$ for some other A.

Consider 3 cases:

Case I. $P_1(A,\bar{B})$ and $P_2(A,\bar{B})$ cross (i.e., there is A such that $P_1(A,\bar{B}) = P_2(A,\bar{B})$) in the region of interest (with ordinate below H and with positive abscissa). Now since $G(A,\bar{Y}) \ge P_1(A,\bar{B})$ for some A, it must cross P_1 in two

places or be tangent to it. Degenerate cases with one or no crossings can easily be proved or eliminated. Let $P_1(A^1, \bar{B})$ be the crossing where the slope of $G(A^1, B) \geq \lambda_1$ (This is the crossing whose abscissa is smaller.). Similarly, for P_2 let $P_2(A^2, \bar{B})$ be the crossing where the slope of $G(A^2, \bar{B}) \leq \lambda_2$ (the crossing with the larger abscissa). Since G is convex in A, G is above the line connecting A^1 and A^2 for $A^1 \leq A \leq A^2$:

$$G(A,\bar{Y}) \geq \frac{P_2(A^2, \bar{B}) - P_1(A^1, \bar{B})}{A^2 - A^1} (A-A^1) + P_1(A^1,\bar{B}).$$

Also, since G is nondecreasing, the point on G with slope $\bar{\lambda}$, which is $\theta(\bar{A},\bar{B})$, will fall in the interval $\left[A^1, A^2\right]$. Hence

$$\theta(\bar{A},\bar{B}) \geq \frac{P_2(A^2,\bar{B}) - P_1(A^1, \bar{B})}{A^2 - A^1} (A - A^1) + P_1(A^1, \bar{B}).$$

But A^1 and A^2 are unknown, so we provide a bound for the worst cases of A^1 and A^2. For example, $P_1(A^1, \bar{B}) \geq P_1(0,\bar{B}) = H^0$ for this case. In $\left[A^1, A^2\right]$ we obtain

$$\theta(\bar{A},B) \quad \frac{H - H^0}{A_0 - 0} (A - A^1) + P_1(A^1, \bar{B})$$

$$\geq \frac{H - H^0}{A_0} (A - A^1) + H^0 + \lambda_1 A^1$$

But in this case $\lambda_1 A^0 \geq H - H^0$. Thus

$$\theta(\bar{A},\bar{B}) \geq \frac{H-H^0}{A^0} (A - A^1) + H^0 + \frac{H-H^0}{A^0} A^1$$

$$\geq \frac{H - H^o}{A^o} \quad A + H^o$$

$$\geq \frac{H}{A^o} \quad A + \left(1 - \frac{A}{A^o}\right) H^o$$

Case II. P_1 (A,B) is everywhere below P_2 (A,B) in the region of interest. That the theorem is true in this case may be seen by noting that this bound is everywhere below the secondary bound in Pugh's paper.

Case III. P_2 (A,B) is everywhere above P_2(A,B) in the region of interest--here the bound is below Pugh's primary bound.*

Notice several things about the procedure implied by the above theorem: 1) many times the primary or secondary bound will be better than the above theorem; 2) since any two solutions bound the optimal curve for lambdas between them, a range of solutions is needed with bounds generated for adjacent lambdas; 3) the optimal curve will be bounded by the minimum of all such single bounds generated; and hence 4) the lambdas must be carefully chosen or one bad bound will spoil the cumulative bound.

The remaining problem is how to apply the verification procedure. The procedure uses a number of Lagrangian solutions for different lambdas. For each lambda it is not necessary to match the desired cost exactly in order to obtain the lower bound; even rough cost closing can give good results. The upper bound

* Actually this theorem can be derived from the primary and secondary bound and amounts to the most efficient use of the two bounds. It is felt, however, that the lemma and the geometric proof (if not the analytic version) are sufficiently intuitive to be important in their own right.

requires interpolation between two solutions to get the bound for the desired cost. There is, however, a more essential difference between upper and lower bounds.

Each upper bound line bounds the entire optimal curve, and hence the bound is the lower envelope of all the bounding lines, a very simple procedure. And for a bound at any one point, a single line may be sufficient.

To do the lower bound, it should be observed that since each pair of lambdas (and these are "adjacent" lambdas) bounds only that portion of the optimal curve between the two lambdas, the entire lambda range (or at least a reasonably large portion) must be covered. And each bounding line so found must be from well-chosen lambdas, for one "bad" line can spoil a bound carefully generated over most of the range. This is because the lower bound is the lower envelope of all lower bound lines generated.* And because of this, the lower bound is the one for which solutions must be carefully chosen to keep the required number of solutions down.

The following method for choosing the lambdas to find the lower bound is suggested; results obtained with it were good. In Figure A-3, suppose that we have a solution for λ_1 giving the line P_1 and resultant attack level A_1. Now if we pick a slightly different λ_1 we may imagine that the new line

* Although it looks like a mistake for both upper and lower bounds to be the minimum envelope, these statements are simply logical implications of the two bounding theorems.

P_2 could pass through the point $P_1(A_1, B)$*; in this case that the worst error (difference between generated lower bound and upper bounds formed by P_1 and P_2) will be at A_1.* If we have some error criterion ϵ, we may mark that below the point $P_1(A_1, B)$, and draw the desired lower bound starting from the intercept H^0 through the point at ϵ below $P_1(A_1, B)$ up to the saturation point

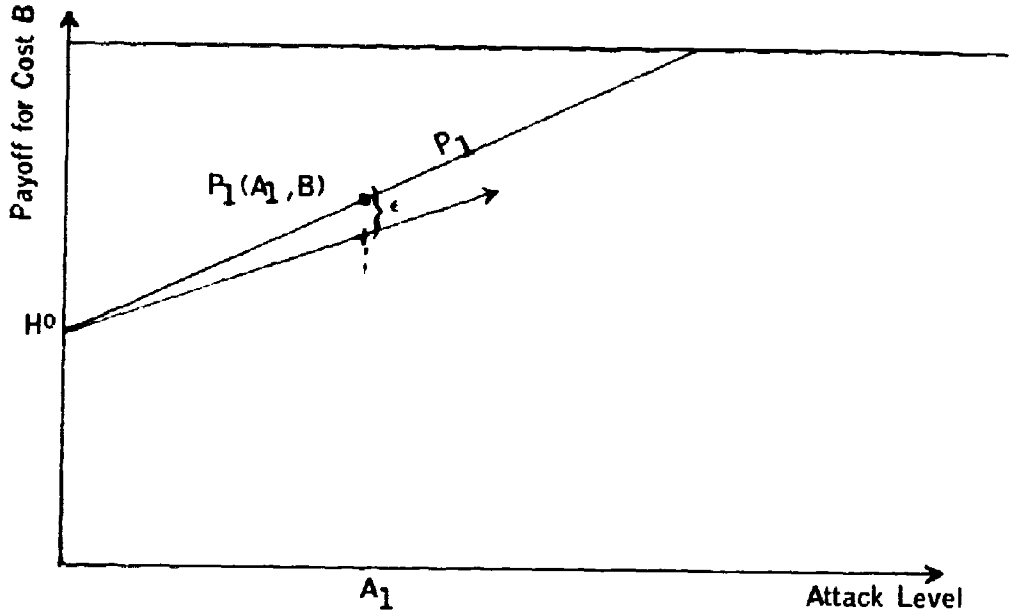


Figure A-3. First Step in Picking New Lambda

A^0. The slope λ_2 is then found by drawing the line from A^0 through $P_1(A_1, B)$, as shown in Figure A-4.

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* Actually if cost closing is exact, P_2 must be above the points; if cost closing is poor, then it will fall below the point.

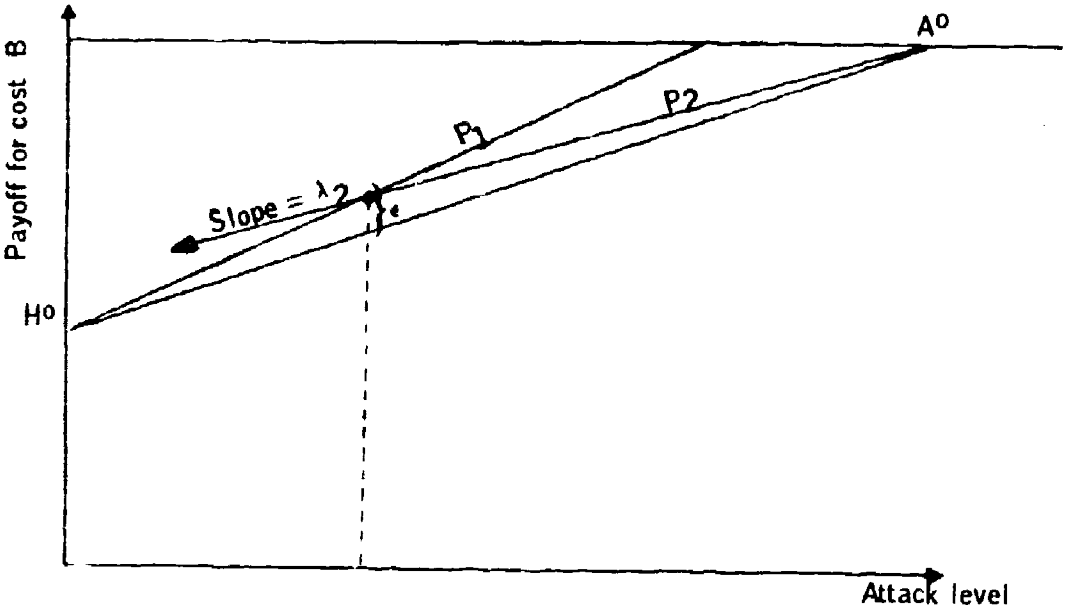

Figure A-4. Chosen Lambda Value

The λ_2 chosen here was less than λ_1. A similar procedure starts from the saturation point of P_1 (instead of H^0) to find higher lambda values.

The procedure shown graphically here can be described analytically, of course, but the problem is merely one in similar triangles, and it will not be presented here. In this procedure, the difference $\lambda_1 - \lambda_2$ is approximately proportional to $1/\epsilon$, and since the entire lambda range must be covered, the error reduction is proportional to 1/(number of Lagrangian solutions).

APPENDIX B

A CITY TARGETING MODEL UTILIZING SIMULTANEOUS WEAPON LAYDOWNS

Loren A. Benson

APPENDIX B

I. INTRODUCTION

The cell model described in the basic report makes feasible the solution of large-scale weapons effects problems which otherwise are intractable. The cell model rests on several assumptions. A targeting model has been developed which, in addition to its general utility as a targeter, helps to illuminate the nature and degree of the approximations of the cell model. This paper describes the targeting model and compares the results that it produces with those produced by the cell model. It is assumed here that the reader understands the operation of the cell model.

The targeting model communicates with the cell model in the sense that the defenses generated and the weapon-to-cell allocations of the cell model can be used to determine weapon allocations in the targeting model. (The actual method of communicating is discussed in another section of this appendix.) The targeting model uses the cell model to suggest ground zeros, generates a weapon drop to correspond roughly to the number of weapons predicted, and calculates the effects of that drop on a citywide basis. By using the cell model as a guide for initial ground zero selection and making the final adjustments (called here "diddling") only after the entire attack is laid down, the targeting model achieves the advantages of simultaneous targeting, in contrast to the method of sequential weapon drops.

II. COMPARISON OF WEAPON EFFECTS CALCULATIONS

Fatality calculations are based on three factors: distribution of population and shelters, distribution of weapons, and the method of calculating weapons effects for the two distributions given. The cell model treats these factors as follows:

1C. Population is distributed uniformly over the entire cell; shelters are distributed over the entire cell.

2C. Weapons laydown is characterized by the number of weapons allocated for a cell--effects are independent of position within a cell, but depend only upon number per cell. As the cell model is implemented in this study, weapons may be allocated only in discrete quanta which may be less than one weapon per cell.

3C. The only pertinent parameters for effects calculations are the area of the cell and the total expected lethal area of the weapons. The total lethal area is calculated from the effects radius for a single weapon and the expected number of weapons delivered. In addition, it is assumed that weapons effects are confined to the cell in which the weapon is delivered. While not inherent in the cell model, the present calculations also assume that weapons effects in a cell can be calculated from an exponential function which considers only the ratio of delivered lethal area to total cell area.

It should be noted that the assumptions in 3C above are entirely consistent with a drop of one weapon which covers more than one cell, since the allowed quantum levels may permit fractional weapons in a single cell. Figure B-1 shows, for example, the cell model prediction for a single weapon drop on Washington. This is consistent with a drop of one weapon at the juncture point of cells 9, 10, 7, and 6.

It is not unreasonable to expect that the cell model would therefore become an accurate guide for weaoon placement and would thus be a valuable first approximation to the exact targeting. The effectiveness of the cell model in predicting ground zeros could be proved if, and only if, there were some method of predicting the absolute maximum kill

Washington, D. C.

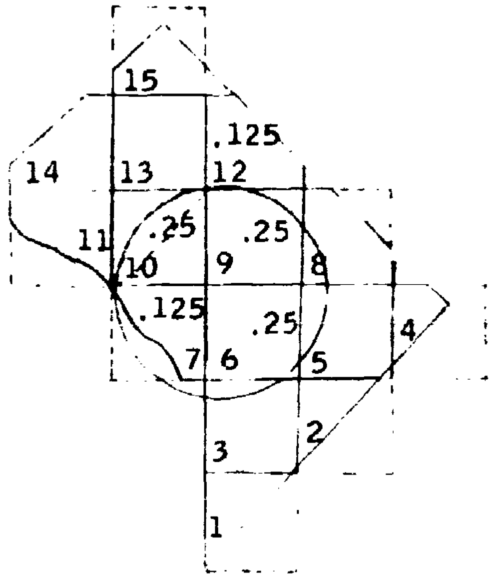

Figure B-1

achievable on a city. Since no theorems have as yet been discovered to find this maximum, the optimality of a targeting doctrine must be determined by heuristic methods. The targeting model program provides a mechanism for taking a particular laydown of weapons, examining it in a systematic manner and attempting to improve the laydown by moving those weapons which kill the fewest people to a location wherein the kill could be increased. This brute force method of moving weapons does not guarantee that the optimum kill configuration will be obtained, but it does guarantee that every weapon move made will improve the kill. If some weapon drop has achieved a local maximum no provision is made for moving two or more weapons simultaneously in order to find the global maximum.

In addition to the possible use of the cell model as a targeting guide, it may also be useful for calculation of fatalities. Its accuracy in this role depends on the applicability of the particular function it employs for cell-wide fatality calculations. The section on results in this Appendix comments further on this question.

In contrast to the cell model, the targeting model is based on the following set of assumptions:

1T. Population is concentrated at points--those points given by the census tracts. For each tract point the population is sheltered proportionally to the shelters allocated for that cell.

2T. Weapons laydown is characterized by ground zeros specified by latitude and longitude.

3T. Weapons effects can be calculated from the yield, CEP, delivery probability, and relative distance of separation.

The first assumption, that of population distribution, may lead to an overestimate of the kill against sheltered population, since the weapons, in either the initial laydown or in the diddling process, may seek local concentrations of population which would not exist in any real shelter situation. Note that the shelter costs are based on some occupancy factor, say 1000 people/shelter; if two or more 1000-person shelters are indicated for a single tract, the kill would be computed on the basis of destruction of more than one shelter's worth at a single point. The actual population distribution is a mixture of shelters, with 1000 people/shelter for the shelter cases and a probably uniform random distribution of unsheltered people within a city. The tract model represents a gross averaging of these effects providing more detail than the cell model but not as much detail as an individual shelter model.

For calculation of weapon effects, the yield, CEP, height of burst, and shelter hardness are all incorporated in a single function, the single shot kill probability function which will calculate the kill of a given shelter hardness at a given distance for a given weapon with a given yield, height of burst and CEP. The calculation of distance between population point and weapon burst point is explicit in the targeting model, although implicit in the cell model. (The cell model also has an implicit interpretation of CEP.) The kill scales in the usual manner with yield; the functions used for calculating the kill closely approximate the sigma 20 and sigma 30 curves for weapons effects.

III. PROGRAM DESCRIPTION

It is important, in understanding the flow of information within the program, to distinguish between two separate interpretations of the targeting model:

1. The model itself represents an approximation to the real world in that the model purports to predict the outcome of a particular sequence of actions, blast shelter deployments, weapon laydowns. The correspondence between the model and the real world (and any real world action) was chosen on the basis of a set of approximations--approximations necessary to allow iterative solutions to the basic equations. This information flow, and model approximation to real world actions, was described in the Introduction to this Appendix, and the rationale for the choice of population distribution representation, weapon effects representation, and weapons laydown approximations was discussed there. All of this discussion forms a preamble to the second description of the model, i.e., its representation in the computer program.

2. For a given set of assumptions, which were chosen on the basis described in Paragraph 1., information is passed from one section of the model to another to attempt to solve the equations. This section of the paper will describe the use of information flow within the model in the solution of the equations.

The computational method suggested by this study to find ground zeros in a city is as follows:

1. Use the cell model to lay down a lambda-constrained attack. From this attack, the weapon density to be delivered to a particular cell is predicted.

2. Using the weapon density found in Step 1. as a guide, make a laydown of actual ground zeros which matches the particular number of weapons/cell. Evaluate such a laydown on a citywide basis. A Monte Carlo selection of ground zeros continues and the set of ground zeros which maximizes the kill is chosen.

3. Diddling, or fine scale improvement, is the final step. An attempt is made to find a better location for the weapons on a citywide basis, and this process continues as long as there is substantial improvement in the kill.

Kill computations may be handled in either of two ways:

1. As an AGZ (no random effects) - the PSI at the point from a weapon at

another point is calculated, and knowing shelter hardness, the kill probability is calculated.

2. As a DGZ - the effects of CEP and delivery probability can be included by using a single shot kill probability function to calculate the expected kill. This use of the DGZ is, of course, the most usual way of treating the problem.

In programming this model an attempt has been made to yield the maximum possible flexibility consistent with the accuracy and speed requirements. This flexibility includes a number of methods of interpreting weapon parameters and shelter parameters. The program is designed to communicate with the cell model, but at the same time act as a completely independent targeting model--receiving its input from outside the cell model and targeting weapons and allocating shelters independently of any information available to the cell model.

In order to allow the user the flexibility described in the preceding paragraph, the program has been written in four separate sections, with control of the various sections monitored by a super monitor program - VALID. One of the subsections, DATAIN, reads (almost) all the data for the program, both parameter descriptions and program flow controls. On the basis of the data inputs through DATAIN,various other sub-monitors are called. These sections are as follows. (See Figure B-2.)

DATAIN - Reads in all the data, prefills the kill computation tables, sets up the monitor controls. The data order is predetermined, and the entire block of data must be read in in DATAIN. (In several of the options additional data is called for; this information always follows the DATAIN package.) DATAIN is always called.

INCELL - This subroutine provides the major communication link with the cell model. At the user's designation the INCELL computation can be bypassed in its entirety, and the model used as a targeter or as an evaluator. In several of the options only INCELL is called, in order to give the user a microscopic view of the cell model--that is, what might happen

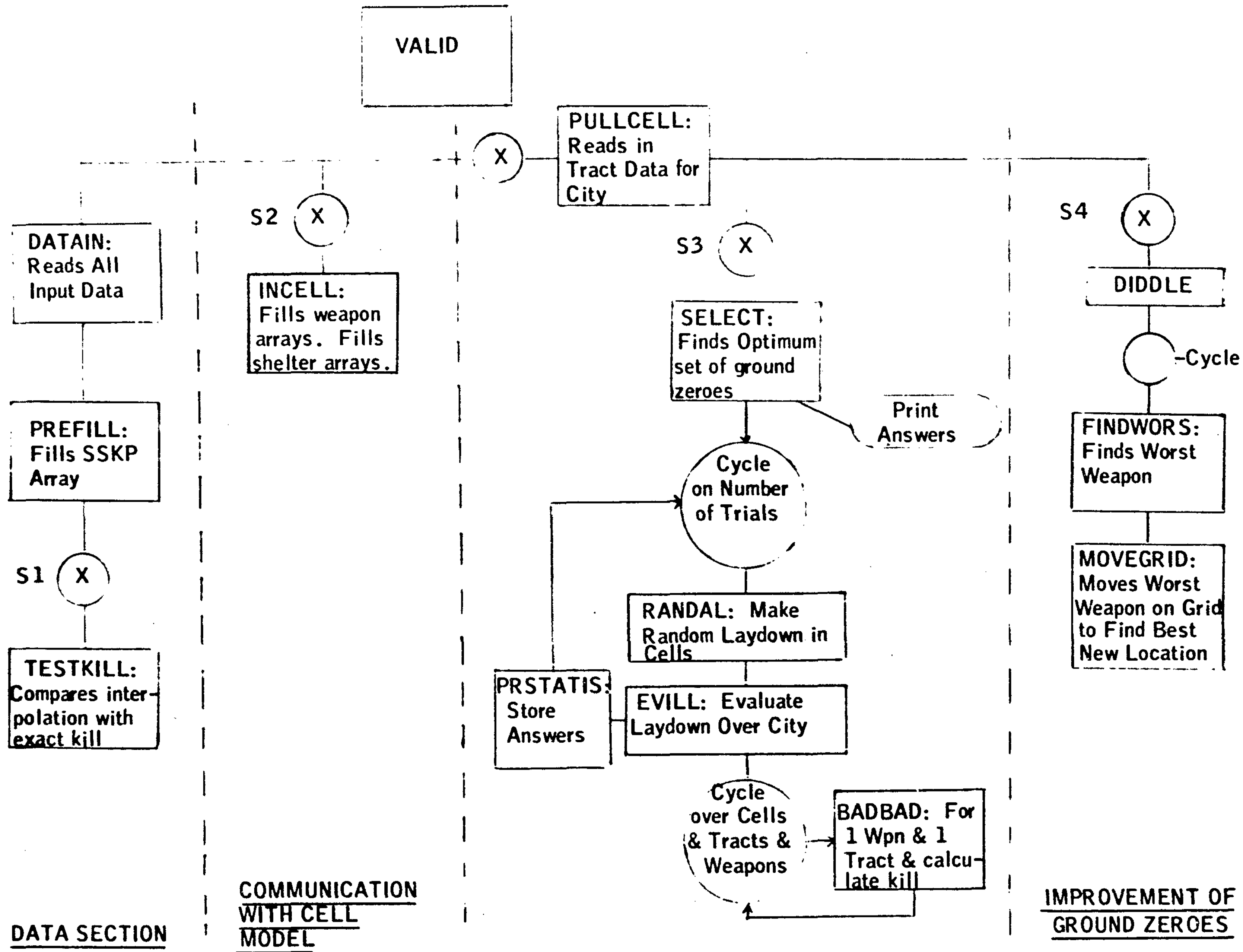

Figure B-2. Basic Optimum Ground Zero Finder

on a single selected city with certain choices of cell model parameters. In certain options, INCELL is called many times for the purpose of choosing a set of parameters for future exploration.

SELECT - This subroutine monitors the basic targeting. It uses arrays either filled from INCELL (from the cell model) or filled from external sources. A fixed point parameter is provided as an argument to the subroutine which could control the targeting algorithm to be used. At the present time, the only path which has been activated is one which does a random drop of weapons within cells and chooses the drop which maximizes the kill. The modular structure of the program allows for expansion to include other targeting doctrines.

The subroutine SELECT (and all of its calls on the targeting programs) may be by-passed. It may be (and has been found to be) desirable to cycle through INCELL in order to determine the effects on a single city. For example, for a number of values of lambda, one might desire to know the number of weapons dropped on each city, and from this list (called a lambda map) select several sets of lambdas and mus for further exploration. A simple switch setting in the data input subsection will allow this cycling.

DIDDLE, the fourth major subsection of the program, incorporates processes to improve weapon allocations. These allocations are based on the output from SELECT, and DIDDLE attempts to move weapons around in an effort to improve the kill. The controls over DIDDLE are rather elaborate--the grid size for weapon moving can be selected, and that to several degrees of accuracy. The control over the cessation of the DIDDLE process depends upon the number of weapons laid down, or some absolute number of weapons, or the amount of improvement which came from the last diddle.
At the end of the DIDDLE cycle, the least effective weapon should have a removal potential roughly corresponding to the lambda used in the initial laydown. For the subset of weapons

with the lowest payoff, the DIDDLE process smooths out the removal potential and tends to equalize all weapons. The DIDDLE process itself should be a fine targeting scheme; however, it is an inherently slow method of doing things. With any laydown, the DIDDLE process should tend to provide an optimal kill, although it does it by selectively placing each weapon on a grid and evaluating the effects of moving that weapon. As a targeter it would tend to be slow and should be used only with a good initial laydown, such as would be provided by SELECT. For speed, the entire DIDDLE process can be bypassed.

The basic process--that one which is repeated many, many times--is the calculation of the kill at a point from a weapon at another point. In the basic laydown, this operation is done for each weapon against each tract; in the DIDDLE process for a single weapon against each tract, this process is repeated for many weapons moved on a grid. Only the pretabling of the kill keeps this computation from being so slow as to prohibit any significant amount of diddling.

The major communication link between subroutines is through information passed through a series of arrays. SELECT depends upon the weapons/cell arrays being filled, as well as the shelter/cell. DIDDLE in turn depends upon the ground zeros being specified (i.e., the latitudes and longitudes) and people/shelter being specified. As long as these arrays are filled, the appropriate monitor subroutine will work, controlling its own subprograms. Communication links between the various monitors is provided by switches set in VALID.

The basic, most time consuming step in the program is the calculation of the kill probability at a given point. Much effort has been devoted to making this calculation as rapid as possible. For a given weapon one computes the kill probability at each population point in the city; if these are tracts, one computes the kill at each of, say, several hundred to several thousand points. For each tract, separate computation must be made of the kill

on each shelter, and there may be as many as five shelters at a given point. The survival probability must be stored for each component at each point. Thus, the kill computation apparently must be made for the product of, first, the number of weapons on a city; second, the number of tracts in a city; and, third, the number of shelters/tract. For a single trial, each basic computation involves the entrance into a function which computes the kill given the weapon yield, CEP, shelter hardness, and distance. Fourth, multiply this basic computation by, say, 100 trials for the basic weapons laydown and the number of basic simple computations becomes horrendous. The DIDDLE process adds yet another dimension since for the best laydown the background kill (defined as the effect of N-1 weapons on the city) is calculated for each of the N weapons. For the worst weapon (that weapon which has the largest background) and a predetermined grid, the kill is computed for each grid point.

In order to expedite this basic kill computation as a function of distance and PSI, the following steps have been taken:

1. The kill is pretabled.
2. At those places where the physics dictates that computation is not needed, it is not made. In this program, a regular grid is laid down over the city. Many of these grid points are tested in the calculation although they lie outside the city limits and a weapon put there will contribute nothing to the kill. It is faster to calculate and discard these points than to precompute that they will be discarded.
   a. If at a given distance in the softest shelter component the kill probability is almost zero, no further kills are made on this tract, since harder shelters will certainly survive.
   b. If population is small (i.e., survivors from other weapons or original filled population), then no kill computation is made on this component.

This pretabling of kill computation accomplishes most of the speedup. The kill computation in the program is made by entering two arrays which have been prefilled. The

first has the survival probability calculated at equal intervals of distance, and the second array has the slope between equal intervals. For a given distance the index to enter the arrays is computed by dividing the distance by the increment used. The survival probability then is simply the survival probability at that distance (found from a single index I) and that hardness (found from a single index J) plus the fractional contribution at t' distance. All of the basic interpolation arrays are precomputed.

The precomputation makes maximum use of the available storage space computing its own distance increment and filling up the entire available space in the following manner. A starting value of Xmin and Xmax (the maximum distance) is given. The number of distance increments (i.e., the amount of storage space) is specified and a starting delta distance is computed. The program then starts with the softest shelter and fills up the kill versus distance table. As the kill is computed,the program examines to insure that at the maximum distance the kill for each shelter component goes to zero. If, for some shelter, at the maximum distance it does not go to zero, the program doubles the maximum distance, computes a new delta and fills up the table. The softest shelter component is that which is certain to be wrong if any are wrong. This doubling process is continued until the survival probability does reach zero. At this point the kill arrays for harder shelters will be filled properly.

An alternative possibility is that the maximum has been selected at too long a distance and the entire available space would not have been used. After the entire table is filled (i.e., for each shelter component) the maximum distance (i.e., for any further distance no kill computations need be made) is known. If the maximum of maximum distances is less than Xmax, then Xmax is set equal to the maximum of maximum distances, insuring

immediately best use of the table. The iteration of table filling generally takes no more than two or three cycles which involves typically one undershoot, a doubling of the distance to produce an overshoot, followed by an exact zeroing in on Xmax, i.e., a rudimentary root finding. In the course of filling the data tables the distance at which the kill goes to zero was determined. This distance is filled into a supplementary table and whenever the kill function is entered the distance is compared with the cutoff distance for this PSI; if the distance is greater than Dmax for that hardness component, a kill of zero is returned and the kill computation further expedited.

After the basic kill versus distance tables are filled up, the program then calculates the interpolation table which is just the kill between successive entries per nautical mile. Thus the difference between the actual distance and the number of increments is multiplied by the kill/mile and added to the basic kill to rapidly produce an interpolated distance.

This scheme, though rapid, is space consuming. It requires two tables for each of two heights of burst (i.e., one interpolation table and one basic table). It also requires a number of distance increments (in this version 60 are used) for a number of hardness components (five in this version); a total of 1200 spaces are required to cover five hardnesses, two HOB's and the entire distance, plus two supplementary cutoff tables--one for each height of burst (i.e., the distance at which the kill for a particular PSI becomes zero).

The more storage space available,the more accurate the interpolation. Since the scheme is a piecewise linear approximation, at relatively short intervals the approximation can be made as accurate as possible by using more space. The interpolation accuracy can be tested directly in the program by calling Subroutine TESTKILL. TESTKILL spans the entire range of distance and delivers the kill as computed directly from the SSKP function and the kill as computed from the interpolated kill function. It selects points at random

from the kill range (sweeping the entire range first from smaller to larger distance and then from larger to smaller so that for the analyst the plotting job is expedited).

The combined effect of the pretabling and interpolation and cutoff in kill computation is sufficient to allow a weapon laydown of some nominal to large number of weapons, one hundred trials and several cycles of diddling, all within an 8-15 minute cycle of computer time suggesting that even tighter coding might speed up the process. For example, the number of trials is presently fixed. However, in many cases the sigma from the first 20 or so trials is only a small fraction of the mean and the casualty distribution is a very tight one. In this case, the appropriate optimum can be achieved with a few trials. This is particularly the case when the population is all soft and the weapons big, and it makes little difference where they are put, since the entire population is killed. If, however, the population is hard, or a mixture of hard and soft, it then may take more trials and more selective diddling to find the correct ground zeros. Thus the program could adjust the number of trials dependent upon the distribution of casualties achieved up to this point.

Tables B-1 and B-2 list the basic data used in the present analysis for computing the result tables in Section V.

Table B-1

CELL	AREA N.MI.^{2}	ORIGIN. POP.	SHELTER OPTION 5	10	20	$ SPENT IN CELL $5	$10	$20
1	2.041	29869	1	2	13	0	1,407,740	7,037,351
2	2.041	30766	1	1	11	0	0	5,416,531
3	4.063	41218	1	1	6	0	0	5,591,910
4	2.041	19341	1	1	2	0	0	1,062,502
5	4.063	106583	2	11	13	3,461,358	12,114,751	17,306,788
6	4.063	127449	11	4	13	15,190,806	7,574,930	21.701,152
7	2.041	120220	10	13	13	2,174,852	3,115,328	3,115,328
8	2.041	22125	1	1	4	0	0	1,874,744
9	4.063	154629	4	2	13	9,129,190	5,230,779	26,153,895
10	4.063	226231	6	13	14	21,694,432	37,452,929	44,368,092
11	2.041	33455	2	10	13	1,341,661	4,683,156	6,708,304
12	2.022	70190	4	4	13	5,720,564	5,720,564	16,388,642
13	4.063	46978	1	6	13	0	6,511,096	11,240,655
14	1.354	39291	4	4	13	2,515,714	2,515,714	7,207,181
15	1.354	22945	1	4	13	0	1,835,841	5,259,438

Table B-2

PEOPLE IN SHELTERS

CELL	DEFENSE = \$5 BILLION PEOPLE AT				DEFENSE = \$10 BILLION PEOPLE AT				DEFENSE = \$20 BILLION PEOPLE AT			
	PSI 10	PSI 42	PSI 141	PSI 424	PSI 10	PSI 42	PSI 141	PSI 424	PSI 10	PSI 42	PSI 141	PSI 424
1	29869	0	0	0	24811	0	5058	0	4578	0	25291	0
2	30766	0	0	0	30766	0	0	0	11299	0	19466	0
3	41218	0	0	0	41218	0	0	0	16932	13878	10408	0
4	19341	0	0	0	19341	0	0	0	15522	0	3819	0
5	94143	0	12440	0	63044	0	43539	0	44384	0	62199	0
6	72855	0	54954	0	88453	38996	0	0	49458	0	77992	0
7	109024	11196	0	0	109024	0	11196	0	109024	0	11196	0
8	22125	0	0	0	22125	0	0	0	12474	9658	0	0
9	107632	46997	0	0	135830	0	18799	0	60635	0	93994	0
10	132010	53841	40381	0	91629	0	134602	0	91629	0	67301	67301
11	28633	0	4822	0	9346	24109	0	0	9346	0	24109	0
12	40741	29449	0	0	40741	29449	0	0	11292	0	58899	0
13	46978	0	0	0	18700	16159	12119	0	6580	0	40398	0
14	26341	12951	0	0	26341	12951	0	0	13390	0	25902	0
15	22945	0	0	0	13495	9451	0	0	4044	0	18902	0

IV. USE OF THE PROGRAM

Almost all of the data required for the targeting program is read in in DATAIN. Shelter specifications and city designations are done in DRIVE. All of the input is contained in a series of arrays, which are used in the various options of running the program. The list of cards to be read is:

1. An alphabetic card which prints out a label for the particular run.

2. The diagnostic cards: many of the subroutines contain diagnostic printing which can be set to print merely by reading in the name of the appropriate subroutine. In many cases this produces voluminous printing which is most useful for debugging. The list of diagnostic controls is terminated by a blank card. If no diagnostic printing is required, the blank card must be included.

3. A card with the basic weapon characteristics is read in. Parameters are yield, CEP, height of burst, and probability of delivery.

4. The set of array fillers follows:

    Array BINPUT, with six elements, interpreted as shown in Table B -3.

    Array ISWITCH, which determines the calling sequence for the various submonitors as well as the interpretation of the parameters. The wealth of attack and defense options can best be understood by examining ISWITCH. Table B -4 gives the use of the various elements in ISWITCH, subsequent tables describe the interpretation of the values ascribed to elements of this array.

    The various values that ISWITCH(1) can assume are given in Table B - 5.

    A value of 6 combined with a value of 2 or 3 in ISWITCH(9) controls special options for reading in defense only and evaluating with lambda. Most useful in evaluating against a stabilized defense, or one not easily derivable from a mu/lambda combination.

    A value of 2, 5, or 6 for ISWITCH(1) causes supplementary cards to be read in: for 2 or 5, weapon and shelter arrays are filled; if ISWITCH(1) = 6 and ISWITCH(10) = 0, then lambda and mu arrays are read in. If ISWITCH(1) = 6 but ISWITCH(10) = 1, only lambdas are read. These supplementary

Table B-3. Interpretation of BINPUT

1	Lambda
2	Mu
3	Switch to control DIDDLE parameters grid control; a value of 1 allows DGZs to be moved over the entire city; 2 restricts the weapon to its cell
4	(Not Used)
5	Total cost of the nationwide defense, used only in the option where total defense cost rather than mu or city defense is read in. If ISWITCH(1) = 7, BINPUT(5) is interpreted as the total number of weapons in the attack.
6	City cost, used in a similar manner to BINPUT(5) to set the defense level on a city, rather than nationwide.

Table B-4. Interpretation of ISWITCH
Sheet 1 of 2

1	Input control switch (see Table B - 5 for explanation of values).		
2	Validation algorithm - currently only a value of 1 is acceptable. If algorithms for weapon assignment other than random within a cell were to be used, the flow in SELECT would be controlled through setting of this switch.		
3	Full validation control -	1	Program cycles through SELECT after choosing initial laydown
		2	Program returns after INCELL, no validation
4	DIDDLE control	1	No fine mesh improvement in DIDDLE subroutine
		2	Fine scale improvement used in DIDDLE
5	Control on filling the kill probability arrays:		
		1	Arrays will not be filled--this option implies that PSI will be calculated directly and speed up provisions will not be used
		2	PREFILL will be called and arrays filled will kill versus distance
		3	Same as 2, but TESTKILL will be called in addition, and interpolation schemes can be compared with direct calculation for accuracy checks
6	Equation to be used in filling arrays:		
		1	Fills kill arrays using PSI
		2	Fills kill arrays using single shot kill probability function, using yield and CEP read in on weapon card

Table B-4. Interpretation of ISWITCH
Sheet 2 of 2

7	Control on tracts or shelters - (Not presently used) - This option is inserted to allow future expansion of the program to go to the fine detail of including individual shelters, with a maximum shelter size to be determined by input cards. If the costing is based on 1000 people/shelter, the program can be modified to spread the tracts (which have typically more than 1000 people) into more numerous points with smaller numbers of people/point.		
8	Use of basic point kill calculation routine:		
		1	Use BAD, which calculates kill based on PSI and does not use the pretabled kill
		2	Use BADBAD which uses the precomputed tables
9	Special control on Input, used in conjunction with ISWITCH(1). If, and only if, ISWITCH(1) = 6.		
		1	Read a lambda/mu list (normal interpretation of ISWITCH(1) - 6
		2	Read a lambda list and evaluate in INCELL without changing preread defense options
		3	Read a value of lambda and evaluate using prefilled defense option
10	Option control for normalizing weapons:		
		1	Normalize weapons/cell as found by INCELL rounding to nearest integer
		2	Use actual cell model outputs from INCELL as expected number of weapons/cell
		3	Same as 2 with additional proviso that total number of weapons/cell is constrained to insure total number of weapons is within one weapon for all trials

Table B-5. Interpretation of Values of ISWITCH(1)

1	Input value of lambda and mu, and determine attack and defense from these values.
2	Input fixed point numbers for numbers of weapons, height of burst, and number of people in each shelter, bypass INCELL computation in its entirety.
3	Input value of lambda and cost of a defense on a single city (BINPUT(6)).
4	Input value of lambda and the nationwide defense cost (BINPUT(5)).
5	Read floating point number of weapons, height of burst and number of people in each shelter (same as 2, except that weapons now treated as expected number of weapons (ISWITCH(10) should be equal to 2 or 3).
6	Read in an array of lambdas and mus and cycle only through INCELL, no validation, prints out only a list of payoffs for lambda and mu and does not give the details of what happens on each cell.
7	Cuts out detailed cell printing in INCELL, just prints out payoff.

reads occur in VALID itself, not in DATAIN and only after the entire data deck is read in. Thus, the card order is fixed and inviolate and even if data is not interpreted it must be read in. In this way the data deck card order never need be altered.

The next array to be supplied is NFIXED, which controls the parameters of the evaluation. Table B-6 describes this array. DIDDLE consists of calls on two basic subroutines, FINDWORS and MOVEGRID. FINDWORS locates the worst weapon by calculating the payoff if each weapon in succession is omitted from the set of weapons dropped. That weapon which makes the smallest contribution to the total payoff is designated as worst (note that locating and moving the worst weapon, under this definition, does not guarantee that an optimum targeting will be found; it is possible that a weapon found by some other criterion than that above should be moved, since the cells are not actually independent). However, this direct method is used rather than any of the more elaborate algorithms (for example, interchange all pairs, move two or three simultaneously). The background survivors from all weapons except the worst are calculated and prestored, and MOVEGRID moves the worst weapon over the grid to find a place where the kill is improved. The grid can be specified to be confined to that cell in which the weapon appears, or can be over the entire city, this being the more usual case. In a major cycle, a grid is set up by taking the minimum and maximum latitudes and longitudes and specifying the number of intervals to divide this into by NXMAJOR and NYMAJOR. (The minimum and maximum latitude and longitude will be that for a cell or for the city, dependent upon the setting in BINPUT(4).) The number of times this process is repeated (the process being call (1) FINDWORS to locate the worst weapon and (2) MOVEGRID to find the best location) is controlled by several other parameters which are explained below. Following completion of the major cycles, the program may undergo minor cycles if NFIXED(8) = 1. In a minor cycle the worst weapon is found using FINDWORS; MOVEGRID is called to find a better place (exactly as was done in a major cycle) but then a fine grain improvement is made. At the best grid point, a smaller grid point, the number in each direction being given by NFIXED(6) and (7) is set up, the size being given in FLOAT. And MOVEGRID locates the final coordinates for the best ground zero. The sequence is

MAJOR CYCLE: FINDWORS, MOVEGRID (repeat until end)

MINOR CYCLE: FINDWORS, MOVEGRID (reset grid parameters for fine mesh, MOVEGRID, repeat until done)

Controls on cycling are given in FLOAT which is described in Table B-7. The last set of input cards are read into FLOAT, an array of 24 elements. SCALE, SCALE2, and SCALE3 can be used in the grid in DIDDLE to insure that the grid chosen is not regular. SCALE is a number between 0 and 1, and should be about .10. A value of .1 will insure that separately the X and Y displacements lie within 10% of the specified (unrandomized) value.

Table B-6. Interpretation of NFIXED

1	NTRIALS	Number of trials in Random Laydown
2	NINTER	Number of intervals in which to accumulate statistics: When making random laydowns of weapons, the program keeps the best value of the payoff, and the first NINTER values of payoff. If NINTER is less than or equal to the number of trials, it orders the total NTRIAL laydown and prints out an ordered list of answers. If NINTER is greater than the number of trials, it uses the first NINTER results to calculate the mean and sigma, and then accumulates the distribution in fractions of sigma intervals below and above the mean
3	(Not Used)	
4	NXMAJOR	Number of grid points in a major cycle in DIDDLE in the X direction
5	NYMAJOR	Number of grid points in a major cycle in DIDDLE in the Y direction
6	NXMINOR	Minor cycle, X direction grid points
7	NYMINOR	Minor cycle, Y direction grid points
8	IFMINOR	Include both major and minor cycles (1=yes, 2=no)
9	NNLAM	Number of lambdas to be read in if ISWITCH(1) = 6
10	NNMU	Number of mus to be read in if ISWITCH(1) = 6

Table B-7. Interpretation of FLOAT

1	(Not Used)	
2	FTOTAL	Total number of major cycles in DIDDLE will not exceed FTOTAL* total number of weapons
3	DXMINOR	Grid size in nautical miles in X direction in minor cycle
4	DYMINOR	Y direction grid size
5	FSECOND	Total number of minor cycles will not exceed FSECOND* total number of weapons
6	FIRMAX	Absolute number of major cycles (cutoff is Min(FTOTAL* total weapons, FIRMAX))
7	SECMAX	Absolute number of minor cycles
8	SCALE	Major cycle scale factor to determine random placement of ground zeros
9	SCALE2	Minor cycle scale factor - major grid
10	SCALE3	Minor grid on minor cycle scale factor
11	XMIN	Minimum X2 for SSKP table: For speed the SSKP table uses distance squared rather than distance for the argument. XMIN is typically set equal to some small number (i.e., .0001 miles^2)
12	XMAX	Maximum distance. This parameter must be greater than XMIN, but need not be chosen accurately. Due to the root finding capability of the SSKP routine (PREFILL), the program will adjust XMAX to the value necessary to fill the array. Iteration cycles can be eliminated if the exact value is put in but this iteration is not very time consuming.
13	NSIGMAS	Number of sigmas to compute intervals to store results of random trials.
14-24	(Not Used)	

V. RESULTS

The targeting model was studied for two purposes:

1. To determine the degree of correspondence between the cell model, which neglects overlap, and a model of targeting, using overlap. Two questions arise here: is the cell model a good targeter, and does it provide realistic fatality calculations?

2. To provide a method for rapid and accurate placement of ground zeros within a city in order to determine the damage to that city, using the properties of simultaneous targeting rather than the more conventional but less accurate methods of sequential targeting.

Tables B-8 through B-19* show exemplar cases in which both the cell model and the targeting model lay down weapons in attacks on Washington, when the nationwide budget for these stabilized defenses is $5, 10 and 20 billion dollars. These weapons in the targeting model are assumed to be one megaton; the results of the cell model are then adjusted so that they can also be interpreted as one-megaton weapons.

It can be seen from these tables that the cell model predicts the fatalities in attacked cells quite well, whether the attacked cells were sheltered or unsheltered. The sheltered unattacked cells show little if any kill, indicating that the cell effects are well confined. The only serious discrepancy of the cell model shows up in cases of unsheltered unattacked cells, in which there is of course no consideration of the effects on a cell which was not attacked directly but whose neighboring cells were attacked. Indeed, it can be said that the differences between the total kills predicted by the two models can be traced to differences in this one case.

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* Tables appear at the end of this section.

One method of improving the cell model is to replace the exponential function by one which considers the entire unsheltered fraction of an attacked cell to be killed if the ratio of delivered lethal area exceeds the cell area. This correction would at least predict that the entire unsheltered population of an attacked cell would be killed, whereas at present the cell model predicts only 60% fatalities in an unsheltered attacked cell*.

The tables also show that the process of "diddling," the improvement of the placement of weapons, brings about relatively small increases in fatalities over those predicted by the cell model. This suggests that the cell model is an excellent mechanism for preliminary selection of ground zeros for a simultaneous targeting scheme. In cases in which only a few (five or fewer) weapons are dropped, diddling attempts to achieve optimal coverage and neglects its kill of hardened people, going after the unsheltered unattacked population so that in this case there are larger differences between the two models than in other cases. In cases in which a larger number of weapons is to be laid down, the situation is different. The cell model might well, on the basis of the population distribution, target one cell of sheltered population with, say, six weapons; the diddling process would move those weapons to destroy unsheltered population. If, however, the number of weapons to be used is sufficiently large, and if the population distribution is relatively uniform over the city, the diddling process does attempt to destroy sheltered population as well as unsheltered.

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* When 0.25 weapon is assigned to a cell, the lethal area is approximately four square nautical miles, the area of the cell. The exponential prediction for the fraction killed is then $1 - \exp - (0.25 \times 4)$ or approximately 60¢.

Table B-8

CASE 47 DEFENSE TOTAL 5 NUMBER OF WEAPONS 4

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	394056	638260	686579
UNSHELTERED KILL	369379	617750	679800
UNSHELTERED/ATTACKED	369379	371186	373283
UNSHELTERED/UNATTACKED	0	246564	306517
SHELTERED AT 42 PSI KILL	24675	10784	2937
SHELTERED/ATTACKED	24675	10784	2937
SHELTERED/UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	0	9727	3842
SHELTERED/ATTACKED	0	9727	3842
SHELTERED/UNATTACKED	0	0	0

Table B-9

CASE 45 DEFENSE TOTAL 5 NUMBER OF WEAPONS 7

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	510997	809638	830113
UNSHELTERED KILL	438009	783369	798977
UNSHELTERED/ATTACKED	438009	430611	445948
UNSHELTERED/UNATTACKED	0	352758	353029
SHELTERED AT 42 PSI KILL	50779	19535	22495
SHELTERED/ATTACKED	50779	5374	8335
SHELTERED/UNATTACKED	0	14161	14160
SHELTERED AT 141 PSI KILL	22209	6464	8641
SHELTERED/ATTACKED	22209	6464	8641
SHELTERED/UNATTACKED	0	0	0

Table B-10

CASE 48 DEFENSE TOTAL 5 NUMBER OF WEAPONS 10

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	761191	859182	868224
UNSHELTERED KILL	682633	798598	805236
UNSHELTERED/ATTACKED	682633	699932	701331
UNSHELTERED/UNATTACKED	0	98666	103905
SHELTERED AT 42 PSI KILL	69751	50129	51885
SHELTERED/ATTACKED	69751	50129	51885
SHELTERED/UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	8807	10455	11103
SHELTERED/ATTACKED	8807	10455	11103
SHELTERED/UNATTACKED	0	0	0

Table B-11

CASE 46 DEFENSE TOTAL 5 NUMBER OF WEAPONS 15

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	897259	913351	913351
UNSHELTERED KILL	786863	824613	824613
UNSHELTERED/ATTACKED	786863	805273	805273
UNSHELTERED/UNATTACKED	0	19340	19340
SHELTERED AT 42 PSI KILL	88187	70797	70797
SHELTERED/ATTACKED	88187	70797	70797
SHELTERED/UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	22209	17940	17940
SHELTERED/ATTACKED	22209	17940	17940
SHELTERED/UNATTACKED	0	0	0

Table B-12

CASE 144 DEFENSE TOTAL 10 NUMBER OF WEAPONS 5

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	676510	701624	726399
UNSHELTERED KILL	234498	633514	651486
UNSHELTERED/ATTACKED	234498	266027	240685
UNSHELTERED/UNATTACKED	0	367487	410801
SHELTERED AT 42 PSI KILL	22927	46189	50695
SHELTERED/ATTACKED	22927	38001	30913
SHELTERED/UNATTACKED	0	8188	19782
SHELTERED AT 141 PSI KILL	21713	21921	24218
SHELTERED/ATTACKED	21713	20543	20543
SHELTERED/UNATTACKED	0	1378	3675

Table B-13

CASE 143 DEFENSE TOTAL 10 NUMBER OF WEAPONS 5

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	280095	634084	717842
UNSHELTERED KILL	229928	569718	628407
UNSHELTERED/ATTACKED	229928	225429	215143
UNSHELTERED/UNATTACKED	0	344289	413264
SHELTERED AT 42 PSI KILL	28454	39865	60485
SHELTERED/ATTACKED	28454	39865	39865
SHELTERED/UNATTACKED	0	0	20620
SHELTERED AT 141 PSI KILL	21713	24501	28951
SHELTERED/ATTACKED	21713	24501	24500
SHELTERED/UNATTACKED	0	0	4451

Table B-14

CASE 134 DEFENSE TOTAL 10 NUMBER OF WEAPONS 9

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	676510	796469	812875
UNSHELTERED KILL	601956	726625	733048
UNSHELTERED/ATTACKED	601956	626165	626165
UNSHELTERED/UNATTACKED	0	99460	106983
SHELTERED AT 42 PSI KILL	42509	52789	62773
SHELTERED/ATTACKED	42509	46659	46659
SHELTERED/UNATTACKED	0	6130	16114
SHELTERED AT 141 PSI KILL	32046	17055	17055
SHELTERED/ATTACKED	32046	16886	16886
SHELTERED/UNATTACKED	0	169	169

Table B-15

CASE 132 DEFENSE TOTAL 10 NUMBER OF WEAPONS 14

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	796168	867968	880680
UNSHELTERED KILL	687833	731574	734744
UNSHELTERED/ATTACKED	687833	712351	715403
UNSHELTERED/UNATTACKED	0	19223	19341
SHELTERED AT 42 PSI KILL	72840	88739	98280
SHELTERED ATTACKED	72840	88739	98280
SHELTERED UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	35495	47656	47656
SHELTERED ATTACKED	35495	47656	47656
SHELTERED/UNATTACKED	0	0	0

Table B-16

CASE 140 DEFENSE TOTAL 10 NUMBER OF WEAPONS 16

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	812494	884311	884311
UNSHELTERED KILL	609681	721526	721526
UNSHELTERED/ATTACKED	609681	655626	614409
UNSHELTERED/UNATTACKED	0	65900	107117
SHELTERED AT 42 PSI KILL	97178	85032	85032
SHELTERED/ATTACKED	97178	85032	85032
SHELTERED/UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	105635	77752	77752
SHELTERED/ATTACKED	105635	77752	777[illegible]
SHELTERED/UNATTACKED	0	0	0

Table B-17

CASE 137 DEFENSE TOTAL 10 NUMBER OF WEAPONS 26

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	942451	946579	955380
UNSHELTERED KILL	729213	734862	734862
UNSHELTERED/ATTACKED	729213	734862	734862
UNSHELTERED/UNATTACKED	0	0	0
SHELTERED AT 42 PSI KILL	107600	118596	125794
SHELTERED/ATTACKED	107600	118596	125794
SHELTERED/UNATTACKED	0	0	0
SHELTERED AT 141 PSI KILL	105638	93120	94724
SHELTERED/ATTACKED	105638	93120	94724
SHELTERED/UNATTACKED	0	0	0

Table B-18

CASE 54 DEFENSE TOTAL 20 NUMBER OF WEAPONS 5

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	382615	471329	492015
UNSHELTERED KILL	325776	384318	398926
UNSHELTERED/ATTACKED	325776	352982	349766
UNSHELTERED/UNATTACKED	0	31336	49160
SHELTERED AT 42 PSI KILL	0	0	4153
SHELTERED/ATTACKED	0	0	0
SHELTERED/UNATTACKED	0	0	4153
SHELTERED AT 141 PSI KILL	51944	80235	82169
SHELTERED/ATTACKED	51944	80235	79633
SHELTERED/UNATTACKED	0	0	2536
SHELTERED AT 424 PSI KILL	4895	6766	6766
SHELTERED/ATTACKED	4895	6766	6766
SHELTERED/UNATTACKED	0	0	0

CASE 52 DEFENSE TOTAL 20 NUMBER OF WEAPONS 9

	CELL MODEL	TARGET MODEL	DIDDLE
TOTAL KILL	480669	566521	577392
UNSHELTERED KILL	374522	438345	438480
UNSHELTERED/ATTACKED	374522	395438	395572
UNSHELTERED/UNATTACKED	0	42907	42808
SHELTERED AT 42 PSI KILL	5824	10773	10773
SHELTERED/ATTACKED	90889	10619	10619
SHELTERED/UNATTACKED	0	154	154
SHELTERED AT 141 PSI KILL	90889	107866	113627
SHELTERED/ATTACKED	90889	105805	111566
SHELTERED/UNATTACKED	0	2061	2051
SHELTERED AT 424 PSI KILL	9434	9536	15012
SHELTERED/ATTACKED	9434	9536	15012
SHELTERED/UNATTACKED	0	0	0

APPENDIX C

AN ANALYTICAL MODEL FOR BLAST SHELTER DEPLOYMENT

Robert J. Galiano

APPENDIX C

AN ANALYTICAL MODEL FOR BLAST SHELTER DEPLOYMENT

A. INTRODUCTION

In designing a very detailed computer model to investigate the optimal deployment of blast shelters in the U. S., and their effectiveness in reducing casualties in case of enemy attack, it was noted that employment in the model of certain analytic relationships of fairly general acceptability permitted a simple analytic solution to a useful version of the optimum deployment and national effectiveness problem. While this byproduct is of interest in itself it cannot replace the original model for research purposes, because of certain simplifications required in the analytic model.

In this analytic version, only pure sheltering is permitted (uniformly) in each region and a single optimal air burst height* is implied in the relation chosen for individual blast shelter effect in reducing weapon lethal area. In addition, the 12 density regions listed in the Hudson Institute publication "The Design and Performance of 'Optimum' Blast Shelter Programs," by William M. Brown** were chosen for representative calculations on the U. S.; these were checked against 1960 census density distributions given in another study.***

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* Consideration of ground bursts is discussed later.

** Hudson Institute HI-361-RR/2, June 11, 1964.

*** H. A. Knapp, Institute for Defense Analyses, Research Paper P-194. A recent Hudson Institute paper HI-495-RR, Population Density in the United States Urbanized Areas, March 22, 1965, improved the population density distribution. However, the earlier Hudson Institute 12-cell distribution was retained for the present work.

The treatment is based on a rigorous double Lagrange multiplier optimization technique to generate the defense deployment. Such deployments take into account both the defense cost to deploy blast shelters and the attacker's cost to overcome such deployments. This is in marked constrast to most published treatments of pure blast sheltering. In particular, "balanced defense" deployments, which tend to make diverse regions equally attractive to the attacker (generally to the attacker's first weapon), are not in general "optimum" deployments because they ignore the cost to the defense in providing the deployment. Thus, the cost of defending a given area in a more lucrative target is greater than the cost of defending the same area in a less lucrative target because the population density in the region defended is higher in the former target, while the cost of shelters per person sheltered is generally not sufficiently lower in that same area to compensate. However, the cost to the attacker to overcome either depolyment is the same, a function of his weapons' lethal area only. Therefore, blast sheltering is best deployed from the "inside out" with respect to an ordered target list, since the less valuable areas may not be worthy of attack or defense at all and the most valuable regions tend to be too costly to defend except for relatively light attacks or more extravagant defense deployments. The general model and the simplified analytic treatment both naturally reflect this tendency in generating a defense deployment. The resulting optimal deployments, it should be noted, will not make all areas equally "attractive" to the attacker. This variation from "balanced defense" is specifically treated later.

Throughout the treatment which follows, there is a purposeful bias in favor of blast shelters. Thus, essentially 100% occupancy is assumed for all shelters deployed. In addition, cost studies used in the general study tend to attribute to sheltering programs

nearly double the cost expenditures implied in the following results. This type of variation is equivalent to multiplication of the costs labeled on each curve by a factor of two. The choice of optimistic assumptions tends to set an upper limit on blast shelter effectiveness. Thus, the reader can draw negative inferences with high confidence concerning blast shelter effectiveness. Choice of pessimistic assumptions in this case would have to avoid the many situations which would result in nearly zero effectiveness for the shelter system - insufficient warning, undisciplined population, attack on counterforce systems only, etc. Accordingly, we have chosen to give the shelter system all reasonable advantages when choices of assumptions arose.

B. WEAPON EFFECT MODEL

The "weapon density model" described here is basic to both the general and simplified treatments, but the more general approach can vary the weapon effect model as needed.

Consider weapons of lethal area, W, dropped at random into a region of area, A (> W). The probability of survival of a point within the area to N weapons is

$$S = \left(1 - \frac{W}{A}\right)^{N}$$

If weapon density, d, is defined as N/A (weapons/mi.^{2}), we can write

$$S = \left(1 - \frac{Wd}{N}\right)^{N} \quad \text{(weapon density model)}$$

for the survival probability in a region attacked by density, d, of weapons of lethal area, W. The parameter, N, is to be chosen for best representation. By examination of detailed damage calculations for multiple weapon attacks on cities, it has been found that N = 1 best represents "perfect" weapons (zero aiming and delivery error) and very large values of N best represent other cases. In the latter event, a useful

expression is

$$S = \lim_{N \to \infty} \left(1 - \frac{Wd}{N}\right)^{N}$$

or

(1) $$S = e^{-Wd}$$

Equation (1) has been selected for this treatment.

C. BLAST SHELTER COST AND EFFECTIVENESS RELATIONS

To a fair approximation, the effectiveness of a given psi shelter in reducing attack weapon lethal area is proportional to 1/P (where P is the psi hardness of the shelter). For weapons of 1 MT yield, the relation for optimal air bursts between lethal area, W, and psi hardness of the shelter, P, is approximately

$$W = \frac{160}{P} \text{ .sq. mi.)}$$

For other than optimal air bursts, the inverse proportion seems unacceptable, although Hudson Institute used 150 /P (in sq. miles) as a compromise between optimal air burst and ground burst. We choose simply

(2) $$W = \frac{K}{P}$$

for our derivation, and present results for K = 160. "Balanced defense" results presented later were all recalculated for K = 160.

For simplicity, and to permit comparison with their results, the expression for blast shelter cost was taken from the Hudson Institute publication * noted above. The cost relation (dollars/person sheltered) chosen by Hudson Institute was

$$C = 50 + 20 P^{1/2} .$$

* Hudson Institute, op cit.

We have used that relation, as follows

(3) $$C = a + bP^{1/2}$$

These relations, (2) and (3), are admittedly imprecise, but probably as acceptable as any simple analytic functions that could be adopted. Their use here permits comparison of the present results with those of the Hudson Institute "balanced defense" results.

D. POPULATION DENSITY DISTRIBUTION

The following urban population density distribution for the U.S., derived by Hudson Institute, is convenient to use.

Table C-1. Urban Population Density Distribution

ρ_i (density	n_i (number of people)
80 (thousands/sq.mile)	2 million
40 " " "	4 "
20 " " "	6 "
15 " " "	4 "
10 " " "	6 "
8 " " "	7 "
6 " " "	8 "
5 " " "	10 "
4 " " "	15 "
3 " " "	20 "
2 " " "	10 "
1 " " "	4 "

$$N = \sum n_i = 96 \text{ million}$$

E. LAGRANGIAN PAYOFF FUNCTION

The payoff to the attacker, H, assuming the weapon density model, is

$$H = \sum_i H_i A_i$$

where A_i is the area and H_i is the payoff per unit area in the i^{th} cell, i.e.,

$$H_i = \mathcal{f}_i (1 - \exp(-d_i W_i)$$

Including the Lagrange multipliers for attacker, λ , and defender, μ , in the usual way, we obtain for the Lagrangian payoff function (the attacker's profit),

$$H_i' = \mathcal{f}_i (1 - \exp(-d_i W_i) - \lambda C_i^A + \mu C_i^D$$

The cost in the i^{th} cell to the attacker, C_i^A, is simply d_i, the density of weapons applied at that cell. The cost in the i^{th} cell to the defender, C_i^D, is given by equation (3) multiplied by the population density in that cell, $\mathcal{f}_i$,

$$C_i^D = \mathcal{f}_i (a + b P_i^{1/2})$$

The resulting equation for H' is

$$H' = \mathcal{f}(1 - \exp(-dW) - \lambda d + \mu \mathcal{f} (a + b P^{1/2})$$

dropping the subscript "i", but understanding that the optimal results in each cell are to be summed for the nationwide totals. The final relation to obtain an equation in d and P only, is provided by equation (2)

$$W = K/P$$

Substituting, we obtain, finally

$$(4) \qquad H' = \mathscr{Y}\left(1 - e^{-dK/P}\right) - \lambda d + \mu \mathscr{Y}\left(a + b P^{1/2}\right)$$

As described by Pugh* , the optimal deployment can be obtained by determining strategies which are solutions to

$$\min_{P} \left\{ \max_{d} \left[H'(P,d) \right] \right\}$$

F. OPTIMAL ATTACK

In the present case, we may derive the optimal attack by differentiating equation (4) with respect to d to obtain the maximum. Differentiating, then, and equating the result to zero, we obtain eventually

$$(5) \qquad d^* = \frac{P}{K} \ln\left(\mathscr{Y}K/\lambda P\right), \quad \mathscr{Y}K > \lambda P$$

$$d^* = 0, \text{ otherwise}$$

It is convenient to define a special defense level from the relation for $d^* > 0$, i.e., $\mathscr{Y}K > \lambda P$. Thus

$$(6) \qquad P_0 = \frac{\mathscr{Y}K}{\lambda}$$

With defense to the level P_0, the attacker is unable to achieve a positive profit and, hence, does not attack. This defense level, P_0, is identical to the deployment

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* Pugh, George E., Journal Op. Res. Soc. America, Aug., 1964.

level selected by Hudson Institute for "balanced defense." The argument leading to this balanced deployment is approximately as follows. The attacker is to be held to "λ" fatalities per weapon regardless of his attack level. If the attacker's λ exceeds $\mathcal{F} W = \mathcal{F}K/P$, he simply doesn't attack. If his λ is less than $\mathcal{F} K/P$, he can attack and achieve even more than his criterion of efficiency (choice of λ) would demand. At exactly $\lambda = \mathcal{F} K/P$, he is indifferent, hence $P = \mathcal{F} K/\lambda$ is a "balanced" defense.

C. OPTIMAL DEFENSE DEPLOYMENT

Substituting d^* into equation (4), we can minimize the expression with respect to P, again by differentiating.

We finally obtain the following general form,

$$(7) \qquad P = P_0 \exp\left(\frac{-\mu K}{\lambda} \frac{\partial C_D}{\partial P}\right)$$

From the relation given before for C_D, we obtain the explicit form

$$P = P_0 \exp -\frac{\mu b P_0 P^{-1/2}}{2}$$

which is easily solved for P by iteration from $P = P_0$ on the right hand side.

Substituting (7) into (4), we obtain the optimized Lagrangian payoff (profit).

$$\frac{H^{*\prime}}{\mathcal{F}} = 1 - \frac{P}{P_0}\left(1 - \ln\frac{P_0}{P}\right) + \mu a$$

To add realism, the above profit should be constrained not to exceed that at which the defender simply does not defend (at zero cost), i.e.,

$$\left(\frac{H^{*'}}{\mathcal{T}}\right) \text{MAX} = 1 - \frac{P_{unsheltered}}{P_o} \left(1 - \ln \frac{P_o}{P_{unsheltered}}\right)$$

Note that the "μ" term drops out, since the defense cost is zero.

H. OPTIMUM DEPLOYMENT RESULTS

Figure C-1 shows the results of calculations using the mathematical formulation described above to generate optimum blast shelter deployments. Population not provided blast shelters are attributed a hardness of 6 psi. Total urban fatalities are indicated on the left scale and attack size along the bottom. Each curve is labeled with the cost of the blast shelter programs in billions of dollars. For comparison with other studies, curves have also been generated for an unsheltered population hardness of 3 psi. In either case, expenditures of $20 billion or more involve some degree of blast shelter for nearly all the population and no difference would be exhibited from the 6 psi case. This region of higher defense cost is discussed below.

Notice that the relation for the optimal defense level involves the attacker's λ, which corresponds to some attack level. Thus, as might be expected, the choice of assumption about the attack level or λ affects the defense which is found -- each defense is optimized for some attack level. In accordance with the aim of presenting the best blast shelters can do, the curves here indicate the performance of blast shelters optimized at each attack level; the performance of a single defense over the attack level range is not considered.

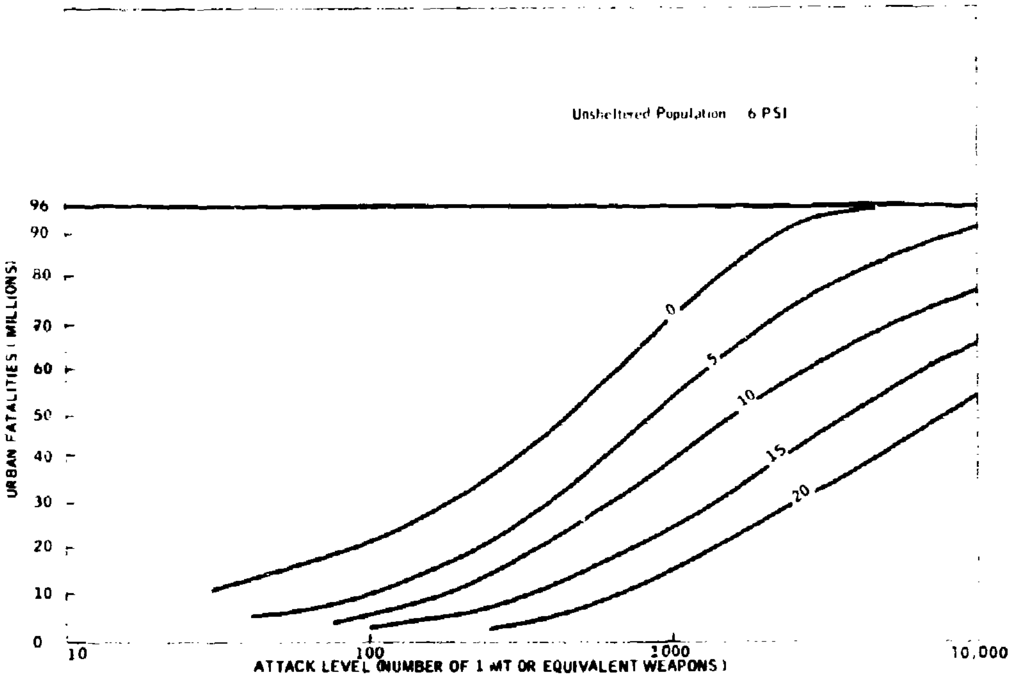

Figure C-1. Effectiveness of Optimal Blast Shelter Programs

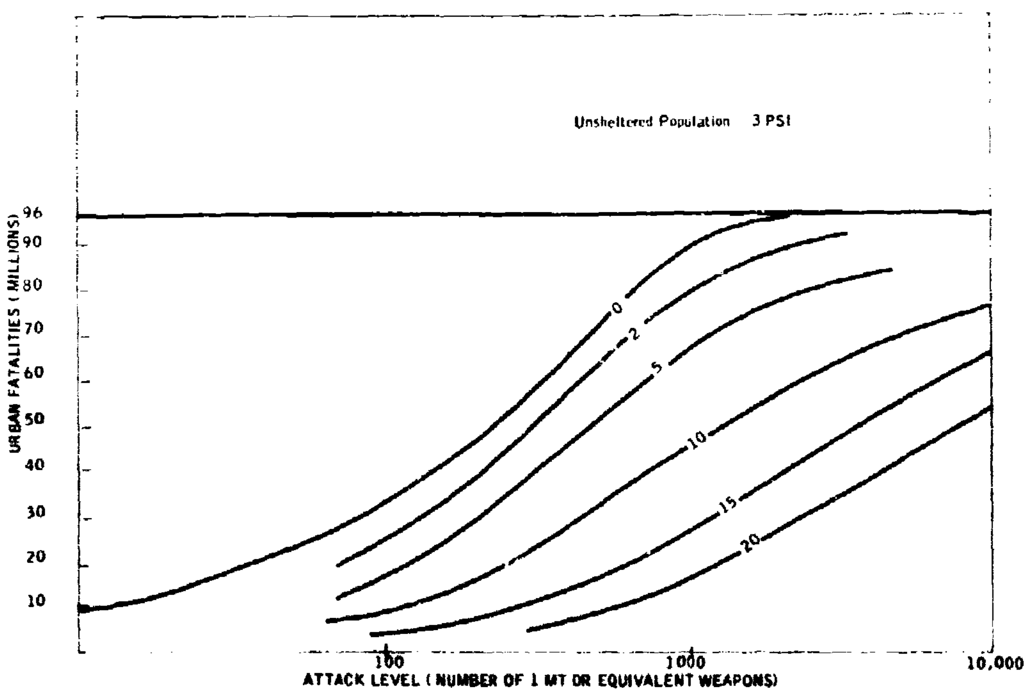

Figure C-2. Effectiveness of Optimal Blast Shelter Programs

I. EFFECT OF LIMITING MAXIMUM SHELTER HARDNESS TO 300 PSI

Since the optimal shelter program often calls for several thousand psi hardness in some regions of high population density, it is interesting to examine the effect on the program of limiting the maximum shelter hardness to a value such as 300 psi, above which construction feasibility and cost information tend to be very uncertain, since additional weapon effects come into play.

In general, the optimum program does not call into use shelters of hardness above 300 psi for program costs less than about $15 billion. At $20 billion, shelters of hardness above 300 psi are used in the few highest density regions alone, and only at lower attack levels; thus, the effect of a hardness limit is still not very noticeable. The effect at $25 and $30 billion is measurable and illustrated in Figure C-3. The hardness limitation sets an upper limit on defense cost at about $38 billion with this model; the higher defense expenditures shown, $35 and $50 and $65 billion were attained with unlimited shelter hardness and cost.

In any realistic program, population outside this target system would become eligible for blast sheltering long before such extreme expenditures that tend to saturate the targets with sheltering.

Another manifestation of the inaccuracy involved in any choice of a limited target set is the artificial suppression of attack efficiency at higher attack levels, when the attacker often is hard pressed to find a defended target lucrative enough to provide the desired payoff (i.e., "λ") per attacking weapon. At the lower end of the target set used for this study, population density is 1000 people/sq. mi. Since the lethal area of a single 1MT weapon is about 27 sq. mi$\left(\frac{160}{6}\right)$, the attacker can obtain 27,000

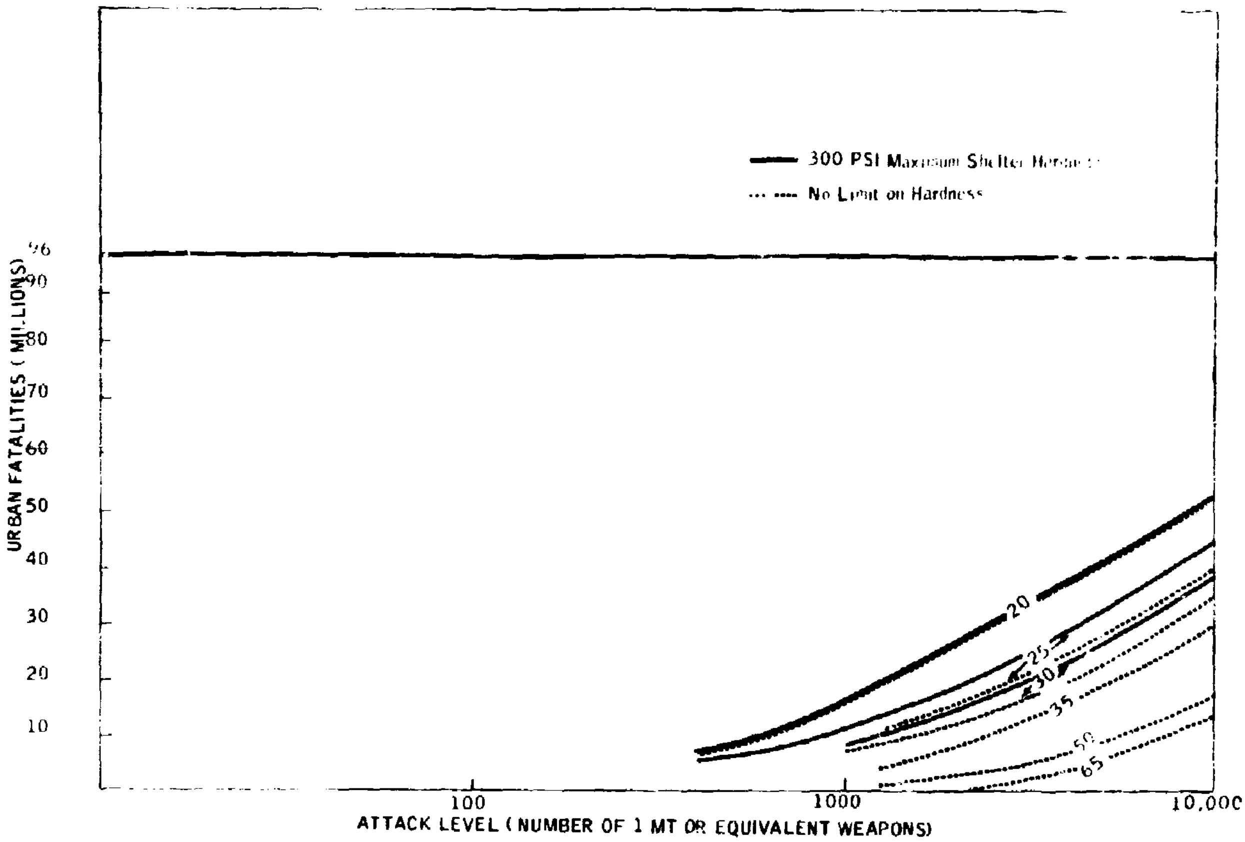

Figure C-3. Effect of 300 psi Limit on Maximum Shelter Hardness

fatalities/weapon in the last cell. However, it is possible that he may obtain even more than 27,000 fatalities per weapon by attacking in regions outside the 12 cells of this study. Thus, optimal attacks generated over the restricted target set for the lower values of "λ" tend to be inefficient to the extent that the attacker can go outside the target set and, perhaps, get the desired "λ" fatalities per weapon in undefended regions. Such regions will lie among the cities of less than 50,000 population outside urbanized areas, which were not included in the formation of the above density distribution.

Assuming such small communities will generally have areas of less than 10 sq. mi., they will therefore often have population densities exceeding 5,000 people/sq. mi.

Thus, the attacker may find lucrative targets outside the five lowest density cells in the target set. Against such cities the attacker may achieve up to 50,000 fatalities per weapon. Communities of 10,000 to 50,000 population outside urbanized areas average about 2,400 people/sq. mi. and include about 16,000,000 people*. The average payoff would approximate 25,000 people/weapon, the same as in the lowest density cell if undefended. Whether any attacker would divert his attack to such non-urban areas is somewhat academic, since there is substantial doubt that such high countervalue attack levels themselves would ever be obtained or sought by an attacker.

J. VULNERABILITY OF UNSHELTERED POPULATION AND BALANCED DEFENSE RESULTS

A critical variable in evaluating the effectiveness of various shelter programs is the vulnerability associated with the unsheltered population. Both 3 psi and 6 psi were used in this calculation, although the latter seems much more realistic, even highly conservative, since a blast shelter program generally presupposes a nationwide fallout shelter program. In addition, the results are based on 100% occupancy of those shelters that are deployed, implying a warned and disciplined population that certainly could take advantage of any fallout shelters available.

The lower hardness of 3 psi was included in these calculations to permit comparison of the present optimal deployments with "balanced" deployments such as those presented by Hudson Institute**, whose "unprotected population" case corresponds

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* H. A. Knapp, op. cit.

** Hudson Institute, op. cit.

fairly closely to the results of this model with a population hardness of 3 psi.*
The optimal blast shelter defense results based on 3 and 6 psi unsheltered hardness are given in Figure C-4.

Figure C-5 shows balanced defense results for various expenditures on blast shelters (billions of dollars). The unprotected population case is taken directly from the Hudson Institute study cited before. The $10 to $50 billion balanced defense expenditure curves were generated by using the analytic relations employed by Hudson Institute. The dotted curve illustrates the vulnerability of an unsheltered population attributed a hardness of 6 psi. Since the unprotected case of Hudson Institute corresponds roughly to a factor of two greater vulnerability (a factor of two less in number of weapons or total lethal area, which varies in indirect proportion with hardness), the 3 psi results of Figure C-2 were generated primarily to compare with the Hudson Institute results. The importance of the vulnerability associated with the unsheltered population is shown by the poor performance of the $10 billion balanced defense curve of Figure C-5, which actually lies above the 6 psi unsheltered population results over much of the span of attack levels shown. The technique of calculation of fatalities for balanced defense deployments is based on the attacker achieving " λ " fatalities per weapon delivered. The defense is "balanced" when the attacker gets just " λ " as a maximum number of fatalities per weapon regardless of the size attack. The technique

* Actually, the Hudson Institute unprotected population curve (Figure C-5) corresponds to a mix of hardnesses generally somewhat above 3 psi. The lethal area relation used here, 160/P, underestimates lethal area in the region of very low psi.

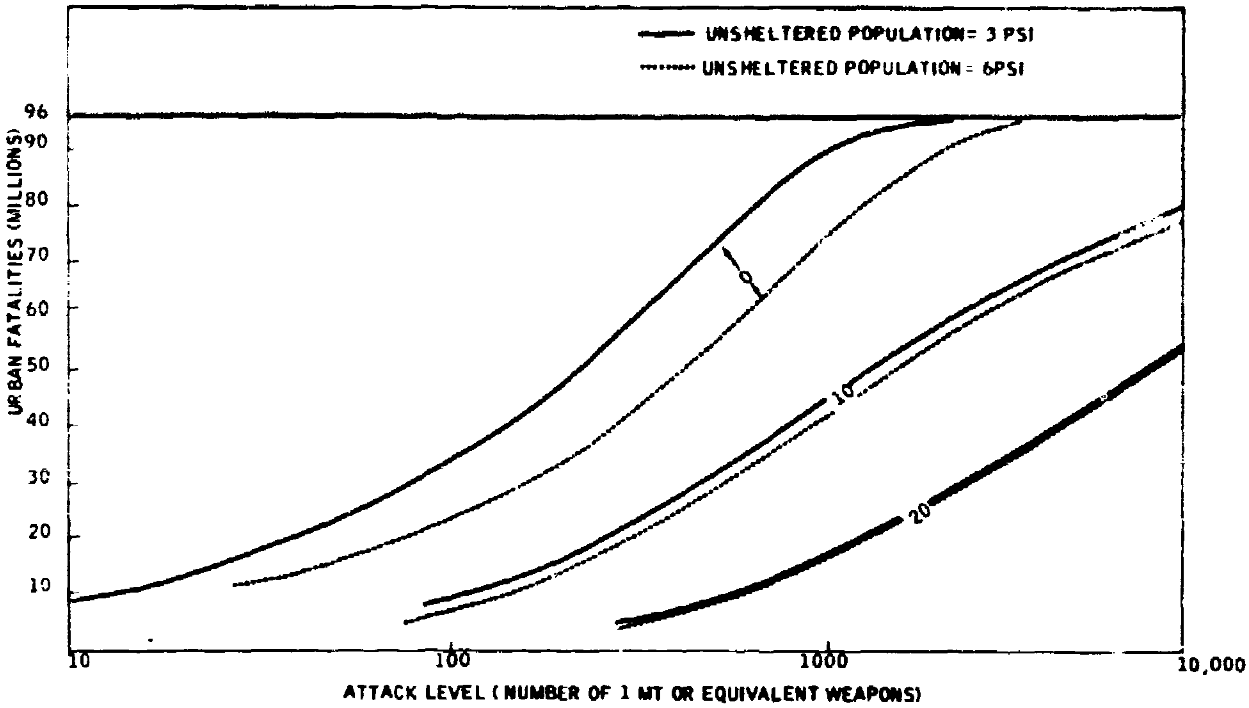

Figure C-4. Comparison of Results for 3 psi and 6 psi Cases

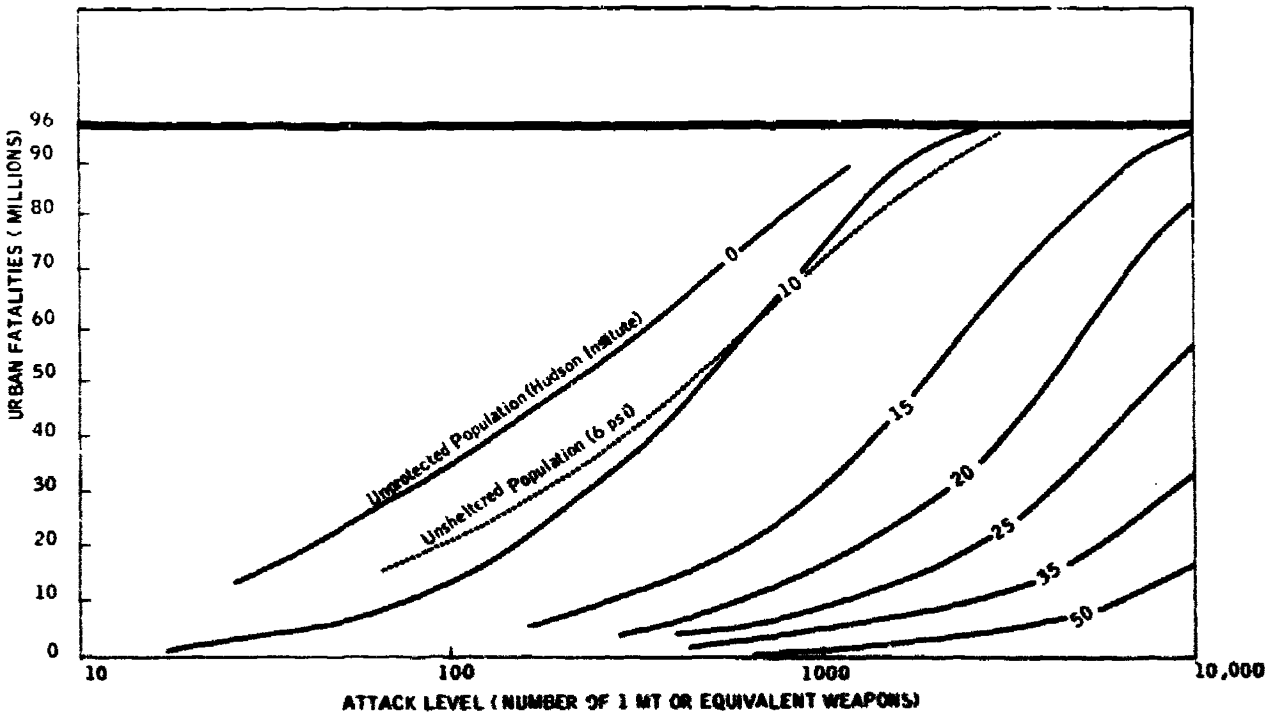

Figure C-5. Balanced Defense Results (Hudson Institute Technique)

used by Hudson Institute can be considered to attribute "λ" to each attacking weapon for derivation of the balanced defense. Thus, for smaller attacks, the fraction of population killed, F, can be approximated very well by:

$$F = \frac{N\lambda}{T}$$

where "N" is the number of weapons and "T" is the total population. This is the relation that is implied in "balancing" the defense deployment.

However, the more realistic equation

$$F = 1\text{-}\exp\left(\frac{-N\lambda}{T}\right) \tag{8}$$

is used by Hudson Institute for calculation of actual fatalities, hence attributing somewhat less than "λ" fatalities to each weapon at higher and higher attack levels. Using the relations previously derived for optimal attacks, equation (8) can be shown to give the total fatalities for an optimal attack against a balanced defense deployment, when equation (1), $S = e^{-Wd}$, represents attack effectiveness in each unit cell.

Unfortunately, equation (8) often tends to overstate attack effectiveness and underrate defense efficiency, as Figure C-5 clearly demonstrates for 6 psi unsheltered population. In this case, the balanced defense actually tends to attribute an enhanced vulnerability to unsheltered people, so that the attacker can still achieve almost "λ" fatalities per weapon in the lower density population cells. In fact, the attacker realistically cannot achieve the "λ" implied by a \$10 billion balanced defense (about 146,000 fatalities per weapon) in the population cells of lower density. This remains true even

for unsheltered population vulnerability of 3 psi. Thus the defender clearly can do better for the lower defense expenditures than shown in Figure C-5.

A more accurate calculation of the effectiveness of balanced defense is presented in Figure C-6. In this case, limits of 300 psi on the highest shelter hardness and 3 psi on the lowest were set with the intermediate cells balanced as before. Three psi (at zero cost) was assigned to all cells wherein balanced defense would usually dictate a hardness below 5 psi. The defenses derived are shown in Table C-2. For each fixed defense, optimal attacks were generated for a spectrum of attack levels. These more judiciously chosen defenses (dotted lines on Figure C-6) are clearly more effective than those implied in generating the results of the preceding Figure, until the 300 psi cutoff inhibits defense effectiveness (above $20 billion). However, for the very high attack levels, the 300 psi cutoff begins to act in favor of the defense since the optimal developments, as shown in more detail later, tend to drop the highest density cells from the defended set as the attack levels increase, while the strictly balanced defenses (solid lines) always place the heaviest defense in the highest density cells.

Figure C-7 compares optimal and balanced defense results from Figures C-2 and C-6 respectively. At higher expenditures the balanced program for defense begins to approach the optimal program. High defense expenditures imply lower and lower values of "μ" in the double Lagrange multiplier treatment, a region in which the optimal psi approaches the maximum psi more and more closely, called P_0 in the earlier derivation, and the deployment hence approaches the balanced deployment of exactly P_0 hardness for shelters in each density region. This tendency is not strong at all if a 300 psi limitation is placed on maximum hardness, as shown in Figure C-7.

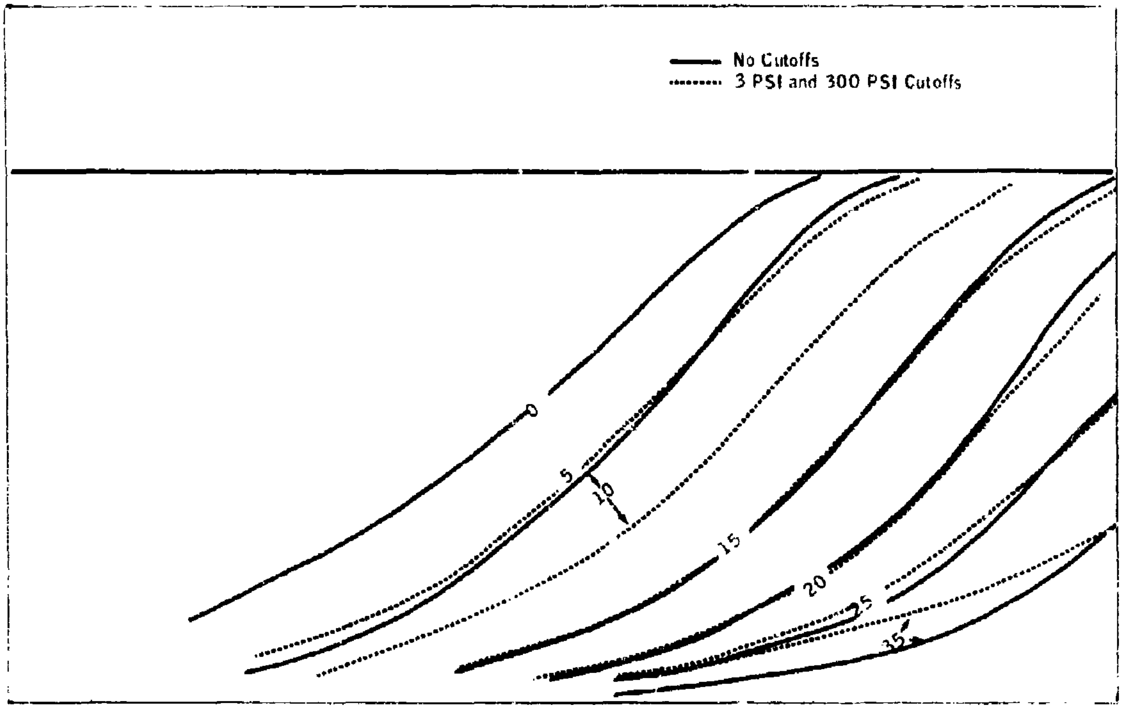

Figure C-6. Balanced Defense Without Cutoffs and With 3 psi and 300 psi Cutoffs

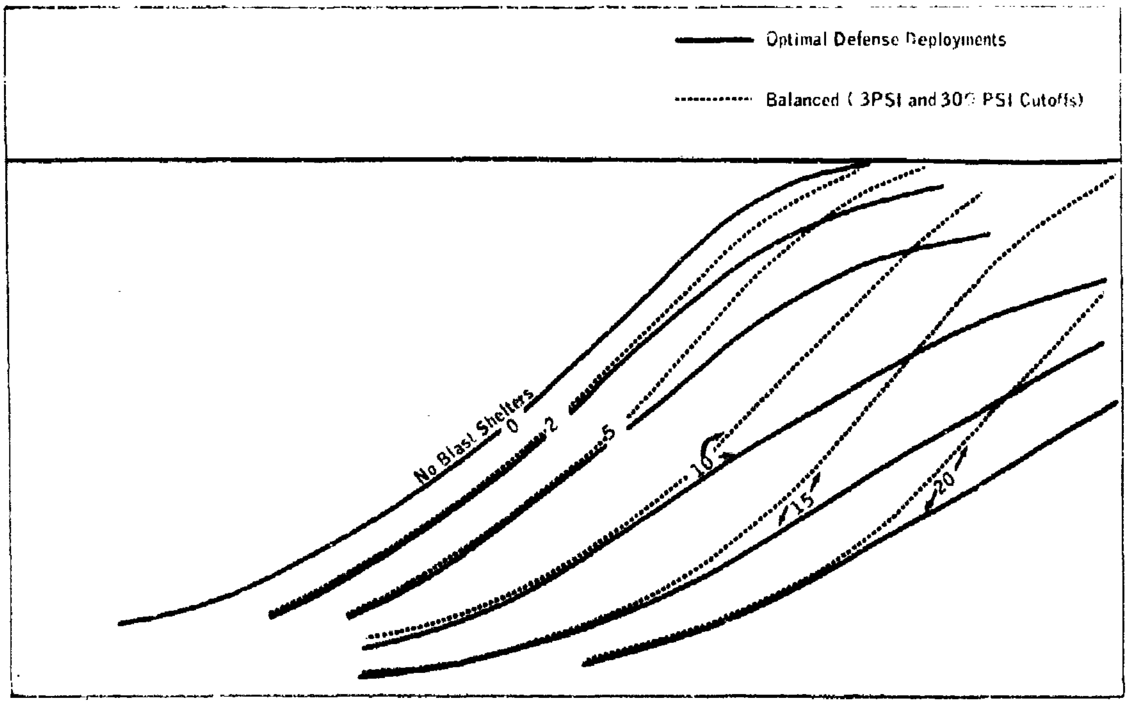

Figure C-7. Comparison of Optimal Defense and Balanced Defense Cutoff at 3 psi and 300 psi

Table C-2. Balanced Defenses
with Cutoffs at 3 psi and 300 psi

Cell Number	Hardness Levels at which Cells are Protected						
	$2 Billion	$5 Billion a/	$10 Billion a/	$15 Billion	$20 Billion	$25 Billion	$35 Billion
1	40 psi	80 psi	132 psi	300 psi	300 psi	300 psi	300 psi
2	20	40	66	184	300	300	300
3	10	20	33	92	205	300	300
4	7 1/2	15	25	69	150	300	300
5	3	9	16.5	46	102	200	300
6	3	8	13	36.5	81	160	300
7	3	6	10	27.5	61.5	120	300
8	3	5 (27%) 3 (73%)	8	22.7	50	100	300
9	3	3	6.5	18	40	80	280
10	3	3	5 (3%) 3 (97%)	14	30	60	213
11	3	3	3	9	20	40	140
12	3	3	3	3	10	20	70

a/ It was necessary to split the last defended cell to achieve the target cost.

The discrepancies toward lower defense expenditures are generally due to overdefense in the highest density cells by the balanced program. As defense is deployed in the optimal program, the middle cells receive the initial increments and the deployment spreads to the other cells of higher and lower population density as more money is spent by the defense. In the present case the last cells to receive defense are those of highest density. The balanced defense, by ignoring the cost of supplying defense (the terms involving " μ " in the payoff formulation),always puts the heaviest defense in the cells of largest population density, where the attacker can most cheaply nullify any deployment. As noted earlier in Figure C-3, $40 and $50 and $65 billion deployments (where balanced defense most closely approaches optimal defense) cannot be achieved except by exceeding a 300 psi maximum hardness. For these higher expenditure regions the defense essentially saturates the restricted target system with shelters - the region of filling out from the middle of the target system in a typical optimal deployment has been passed -- and balanced deployment becomes equivalent to optimum deployment, but only in a mathematical sense, since neither mode can invest the quantity of money implied.

K. OPTIMAL VERSUS BALANCED DEPLOYMENTS

A form of the relation derived earlier for the optimal shelter hardness is useful to show the expected regions of attack and defense level where balanced deployments eventually became equivalent to optimal deployments. This relation was given in equation (7) before,

$$P = P_0 \exp \left\{ \frac{-\mu K}{\lambda} \left(\frac{\partial C_D}{\partial P} \right) \right\}$$

Where C_D is the defense cost function; in the present case,

$$C_D = \mathcal{f}\left(a + b\,P^{1/2}\right)$$

Since balanced defense corresponds to the relation $P = P_0$, the condition for equivalence of balanced and optimal deployments is

$$\frac{\mu K}{\lambda}\left(\frac{\partial C_D}{\partial P}\right) \doteq 0$$

implying $\mu \longrightarrow 0$ or $\lambda \longrightarrow \infty$. The former condition, $\mu \longrightarrow 0$, leads to very heavy defense deployments since the defense is willing to expend an unlimited amount of money to save additional population.* As observed earlier, the very highest expenditures on blast sheltering do lead to optimal deployments equivalent to the balanced deployment. Table C-3 shows this behavior in detail, cell by cell, for three levels of defense expenditure at roughly constant attack level. As noted earlier for the graphical results, the region of equivalence lies at defense costs beyond those which can be achieved with a 300 psi limitation on maximum shelter hardness (about $38 billion). The tendency at $24.7 billion is toward a more uniform distribution of hardness over the population density cells for the optimal deployment, and at $10.7 billion the highest density cells are simply not efficient to defend at all, and the lower density cells are much more heavily defended than for balanced defense. It is interesting to note that balanced defense of $10.7 billion calls for one and three psi hardness in the two lowest density

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* "μ" measures the value the defender places on his blast shelters in terms of required population saved per dollar invested in blast shelters.

Table C-3. Comparison of Various Defense Expenditures on Optimal Versus Balanced Deployments at Constant Attack Level

Cell	Density	Hardness Levels at which Cells are Protected (No Maximum or Minimum Hardness Limitations)					
		1696	Weapons	1554	Weapons	1445	Weapons
Number	Thousands	$48.26	Billion	$24.7	Billion	$10.7	Billion
	Mi^2	Balanced	Optimal	Balanced	Optimal	Balanced	Optimal
1	80	6,100 psi	5,602 psi	1280 psi	785 psi	112 psi	no defense
2	40	3,050	2,917	640	515	56	no defense
3	20	1,520	1,500	320	300	28	no defense
4	15	1,140	1,135	240	235	21	no defense
5	10	760	764	160	165	14	no defense
[illegible]	8	610	614	128	135	11	no defense
7	6	457	464	96	104	8	53 psi
8	5	380	388	80	88	7	46
9	4	305	311	64	71	6	39
10	3	230	234	48	54	4	31
11	2	152	157	32	.37	3	12
12	1	76	79	16	19	1	no defense

cells. As mentioned before, the strict "balancing" process would often require a super-softening below the hardness of the unsheltered population. Thus, the analytic relations developed by Hudson Institute must be used with caution. The more detailed calculation leading to the results presented in Figure C-6 is recommended for any precise analysis.

The other limit at which balanced defense becomes equivalent to optimal defense is $\lambda \longrightarrow \infty$. Since λ measures the attackers propensity to attack,* and varies inversely with attack level, the region of very low attack level, as shown in Table C-4, is the region of equivalence. The general behavior of the optimal defense deployments is as discussed before and requires no special comment again.

L. WHEN IS BALANCED DEFENSE ALSO OPTIMAL DEFENSE?

Aside from limiting cases which may or may not be of practical significance, a version of balanced defense can occur whenever the exponential term is more or less constant with changes in shelter hardness, that is

(9) $$-\frac{\mu K}{\lambda}\left(\frac{\partial C_D}{\partial P}\right) \neq f(P)$$

implying that

$$\frac{\partial C_D}{\partial P} \neq f(P)$$

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* The value he places on his weapons in terms of required (minimum) fatalities per weapon delivered.

Table C-4. Comparison of $20 Billion Defense Deployments Optimized Against Various Attack Levels

Cell	Density Thousands Mi²	Hardness Level at which Cells are Protected					
		Balanced Defense[a/]	150 Weapons	303 Weapons	570 Weapons	2,341 Weapons	5,110 Weapons
		$20.0 Billion	$20.3 Billion	$20.0 Billion	$19.4 Billion	$20.1 Billion	$20.8 Billion
1	80	752 psi	730psi	655 psi	515psi	no defense	no defense
2	40	376	375	349	300	no defense	no defense
3	20	188	191	182	165	205psi	no defense
4	15	141	144	138	127	174	no defense
5	10	94	97	94	88	132	205 psi
6	8	75	78	75	71	111	181
7	6	56	59	57	54	88	150
8	5	47	49	48	46	76	132
9	4	38	39	38	37	63	111
10	3	28	29	29	28	49	88
11	2	19	20	19	19	34	63
12	1	9	10	10	10	18	34

a/ This column shows balanced defense results to compare with the other results to the right, which are all optimal defense deployments at the attack levels indicated.

Thus, any defense cost function that , for instance, is linear with shelter hardness, P, leads to a form of balanced defense. This behavior has been observed before in our studies of the optimization of ballistic missile defense deployments, where the defense effectiveness is approximated by a pure saturation model. Weapons up to a "price of admission" are successfully intercepted by the defense. All excess attacking weapons then penetrate to the target. The chosen function for defense cost was linear with the defense "price of admission" and it was observed that the optimal defense deployments in that case corresponded to balanced defense. Thus, balanced defense cannot necessarily be precluded as non-optimal without further examination of the implicit cost and effectiveness functions used.

It should be pointed out that the balanced deployment is in fact a typical deployment subset of the entire optimal deployment. The balanced region in the optimal deployment corresponds to a region in the solution space where the defense could make the attacker's Lagrangian expression negative. An obvious boundary condition, however, causes a zero value of the attacker's Lagrangian to be in force in any such region. When the attacker's Lagrangian is zero, he can obtain over that subset of the targets only zero profit, and is therefore indifferent between attack or no-attack. In this region also the optimal defense results are, of course, independent of mu, which implies an independence of the defense cost to deploy the shelters. It is essentially a saturation phenomenon.

APPENDIX D

POPULATION AND INDUSTRY DISTRIBUTIONS

APPENDIX D

A. POPULATION AND INDUSTRY DISTRIBUTIONS

The population and industrial data used in the program was sorted by population density and industrial density to obtain several distributions. The first of the distributions are the marginal distributions, the distributions of a factor without considering any of the other factors. For reference, the total daytime population is 209,474,000, total nighttime population is 207,959,000, and total industrial value is $97,263,000,000.

The first marginal distribution is shown in Figure D-1; it shows the amount of daytime population below each population density. For example, according to the graph, 100 million people are located in cells whose population density is less than 4000/persons/sq.n.mi., and a few million people are located in cells whose density is greater than 100,000. The points shown in the figure are the data from which the curve was plotted; such points are not shown on subsequent curves.

Figure D-2 shows the distribution of nighttime population, and Figure D-3 shows the distribution of the maximum population in each cell.* The industrial data also has an associated industrial value density, and a similar plot can be done for the industrial value distribution as shown in Figure D-4.**

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* Note that the total is not the same as the day or night population, since $\sum$max (day population, night population) $\neq$ max (total day population, total night population).

** The program considers explicitly only those cells whose population or industrial density is above some cutoff; all other cells are lumped into 10 "tail cells". These tail cells diluted the industry in lower density brackets, resulting in the long tail in Figure D-4.

The first pair of curves depicting the joint character of population and industry are Figures D-5 and D-6. The first is the average industrial density in areas with a given population density; the second shows the average population density around industry. In each industry bracket the average population density is computed from

$$\frac{\sum \text{Population x Population Density}}{\sum \text{Population}}$$

and similarly for the average industry. The figures show the average densities, the cells in any area may be expected to deviate from the curve. For example, Figure D-5 includes several cells selected from Washington, D. C., Philadelphia, and Chicago to illustrate how the curve is an average of all such cells. Each of the letters plotted represents a cell in that city with population density and industry density as indicated. From these curves we see that areas of highly dense industry, on the average, have dense daytime population. However, in highly dense population areas, the average industry density is relatively low.

Another aspect of the jointness of population and industry is shown in Figures D-7 and D-8. In statistical terms, Figure D-7 shows the cumulative distribution of industry for various population densities; the cumulative distribution of population with respect to population density is shown for comparison. Figure D-7 indicates how much industry is located where the population density is less than some specified level. Similarly, Figure D-8 shows the cumulative distribution of population with respect to industrial density. From Figure D-7, for instance, we can observe that 47% of the population and 31 of the industry are located in areas with less than 5000 persons

per square mile.*

This pair of graphs is useful in that unsheltered fatalities in a pure population or pure industry attack can be determined directly from them. In the two types of attack, the attack density is a function of population or industry density, respectively. Then for different ranges of density the fraction killed is determined, and the population in each different fractional kill bracket is read from the curves. Total fatalities then come directly.

*These last two figures deserve a word of caution: they should not be interpreted as saying more than is indicated. Observe, for instance, that Figure D-7 indicates that half the population is located in areas with 33% of the industry, and that Figure D-8 indicates that half the population is located in areas with 10% of the industry. The explanation, of course, is that it is not the same 104,000,000 people, and the curves in no way indicate that it is the same population.

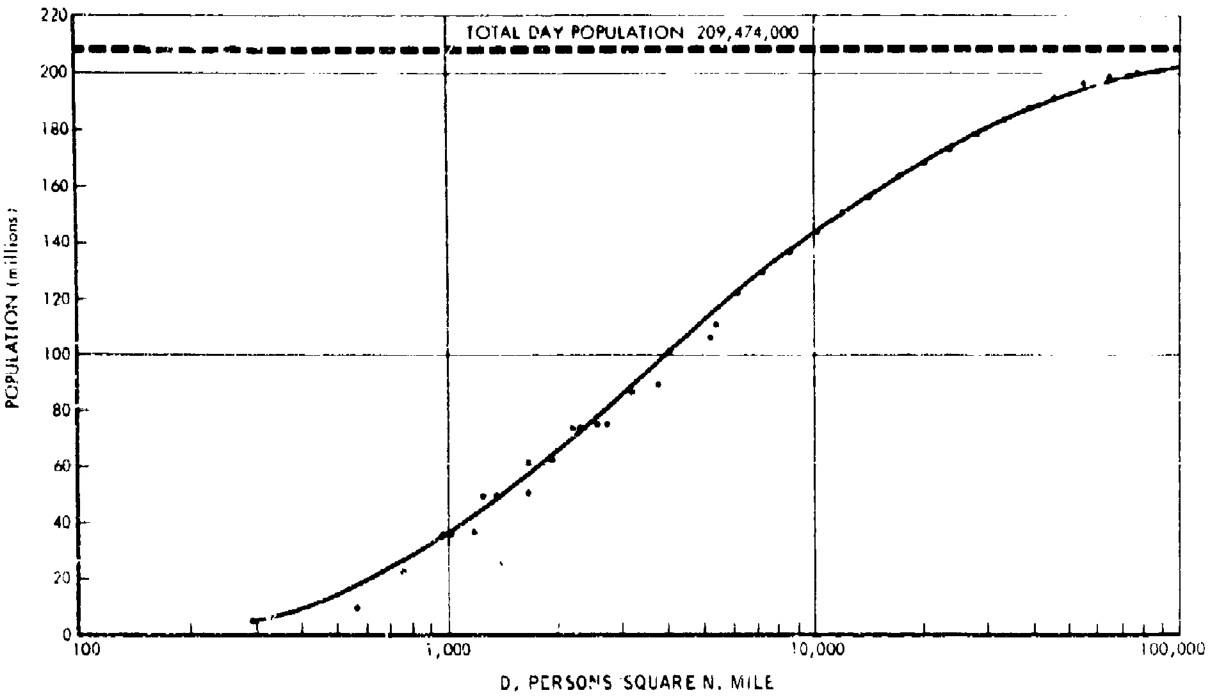

Figure D-1. DAYTIME Population Located Where the Population Density is Less Than D

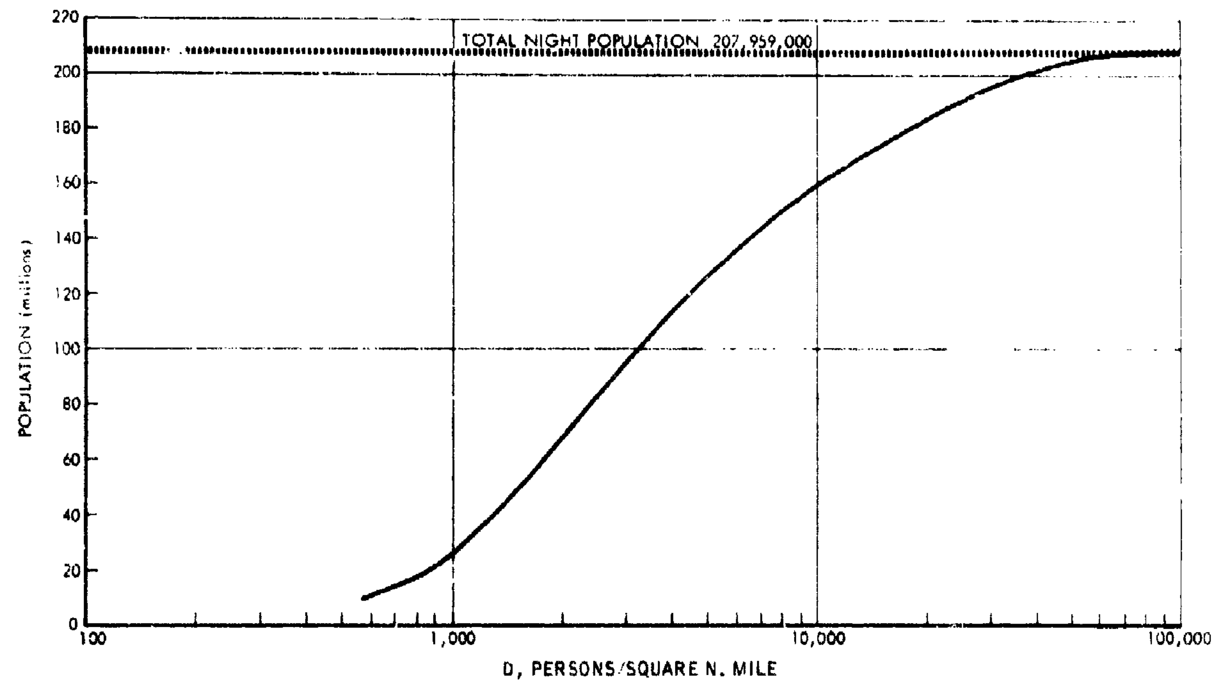

Figure D-2. NIGHTTIME Population Located in Areas Where the Population Density is Less Than D

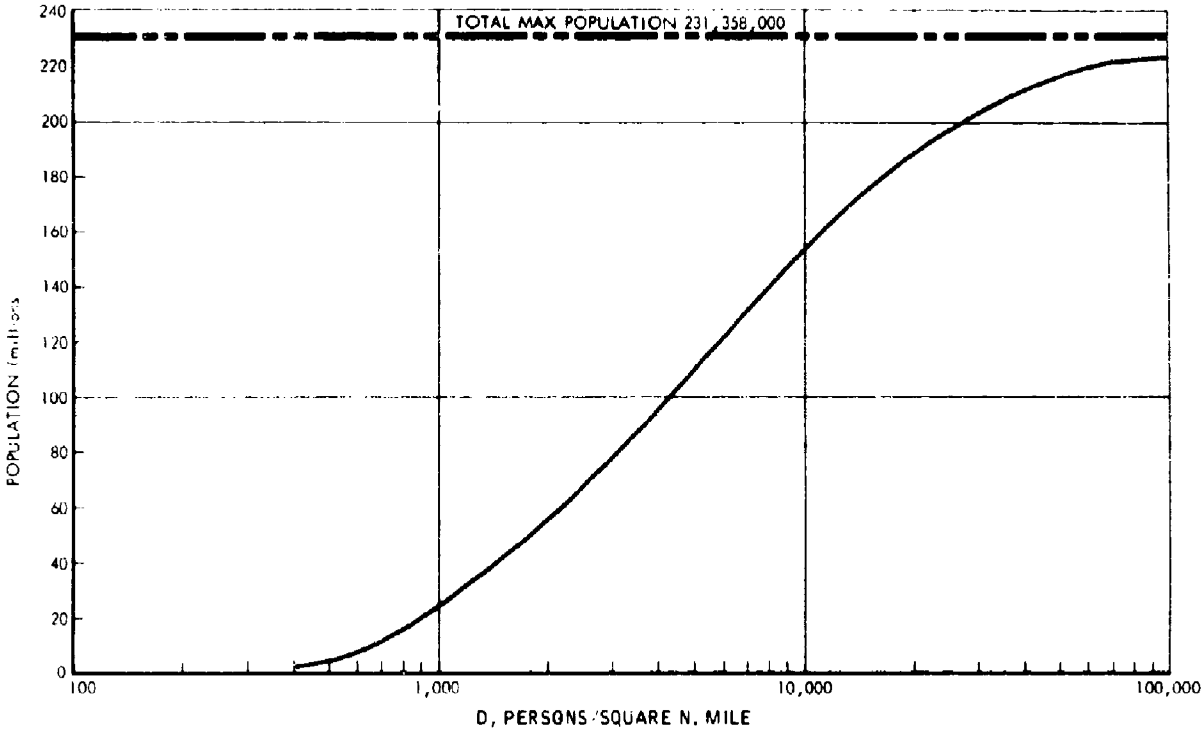

Figure D-3. Portion of MAXIMUM Population Located Where the Population Density is Less Than D

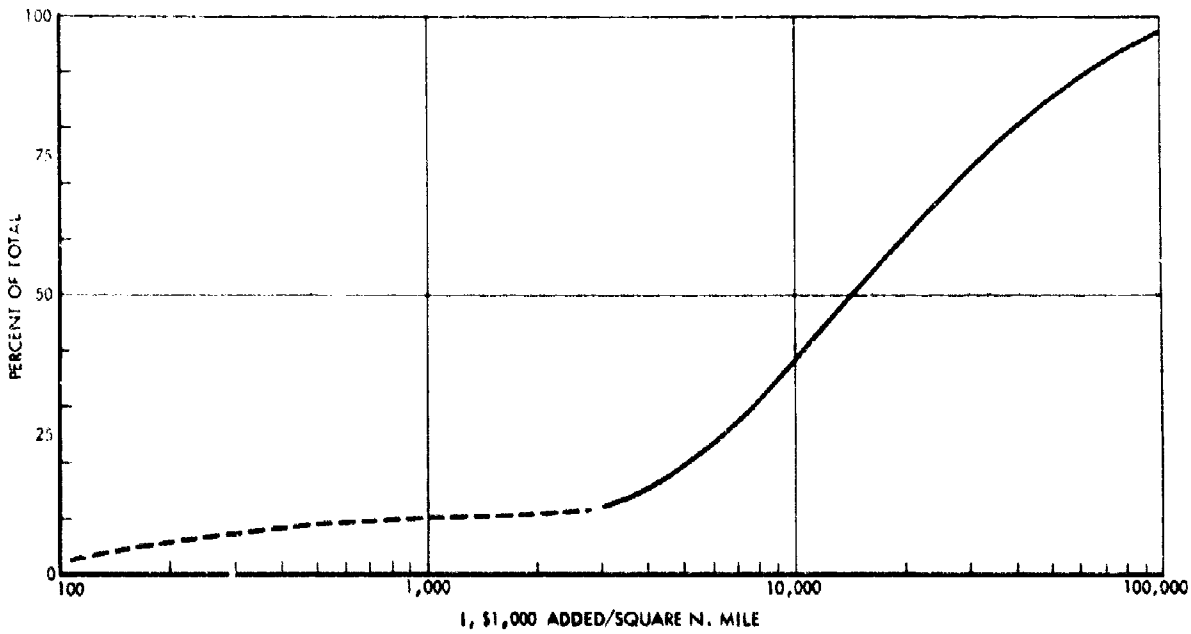

Figure D-4. Percentage of INDUSTRY Located Where the Industrial Density is Less Than I

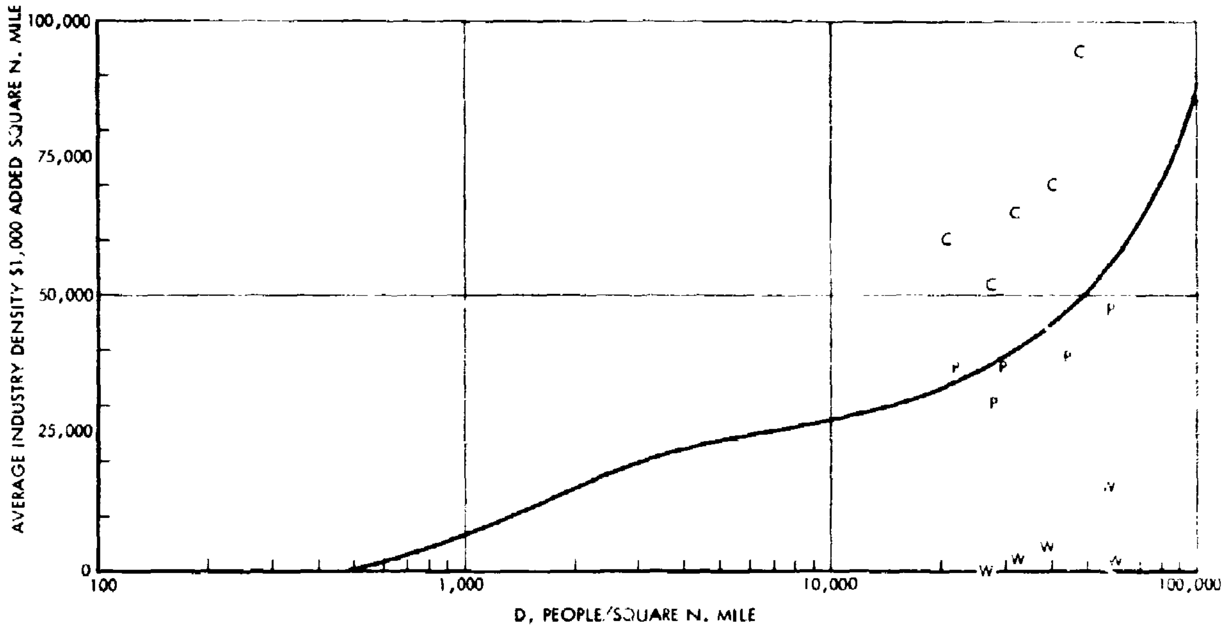

Figure D-5. Average Industry Density in Areas Where the Daytime Population Density is D, Including Several Example Cells from Washington, D.C., Philadelphia, and Chicago

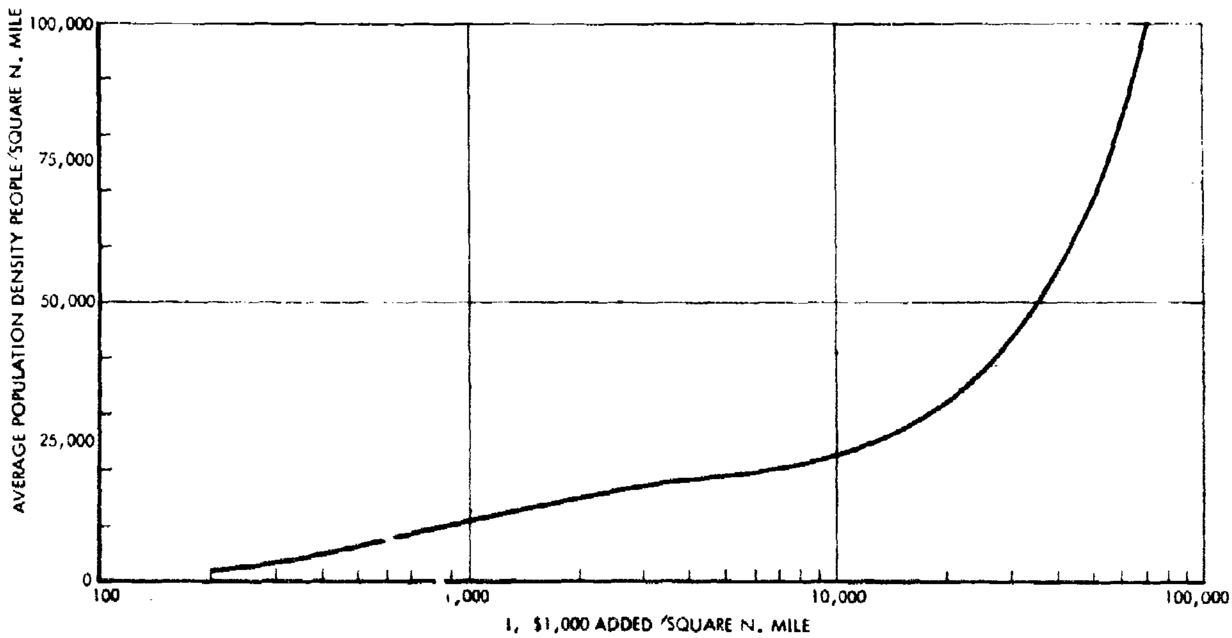

Figure D-6. Average DAYTIME Population Density in Areas Where the Industrial Density is I

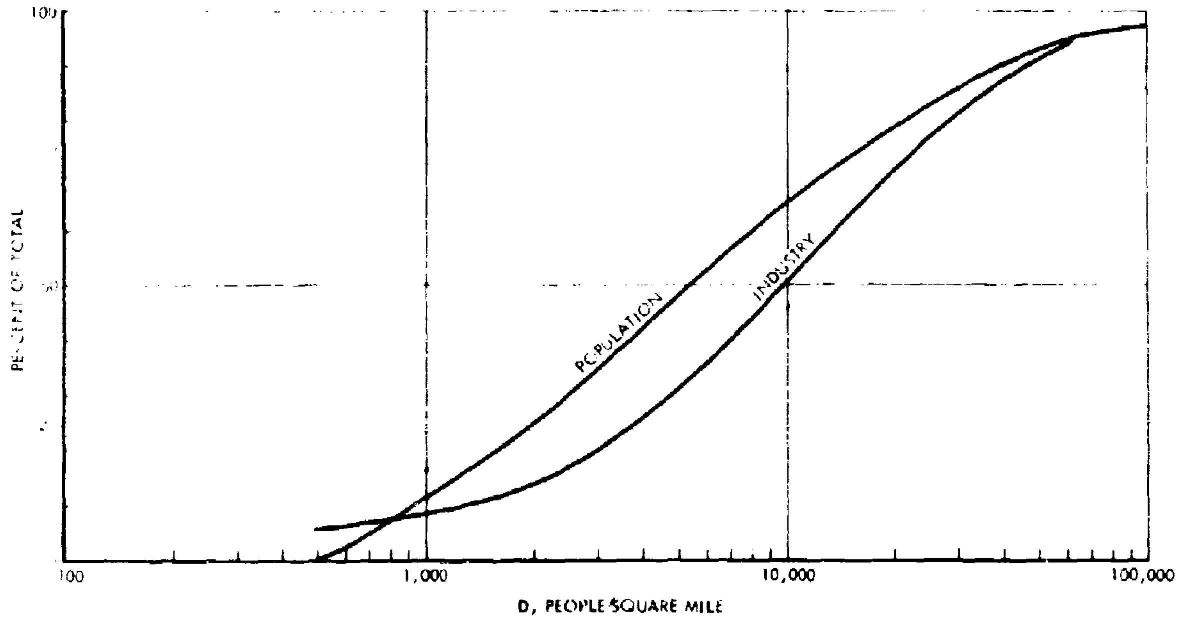

Figure D-7. Percentage of Maximum Population and Industry Located Where the Population Density is Less Than D

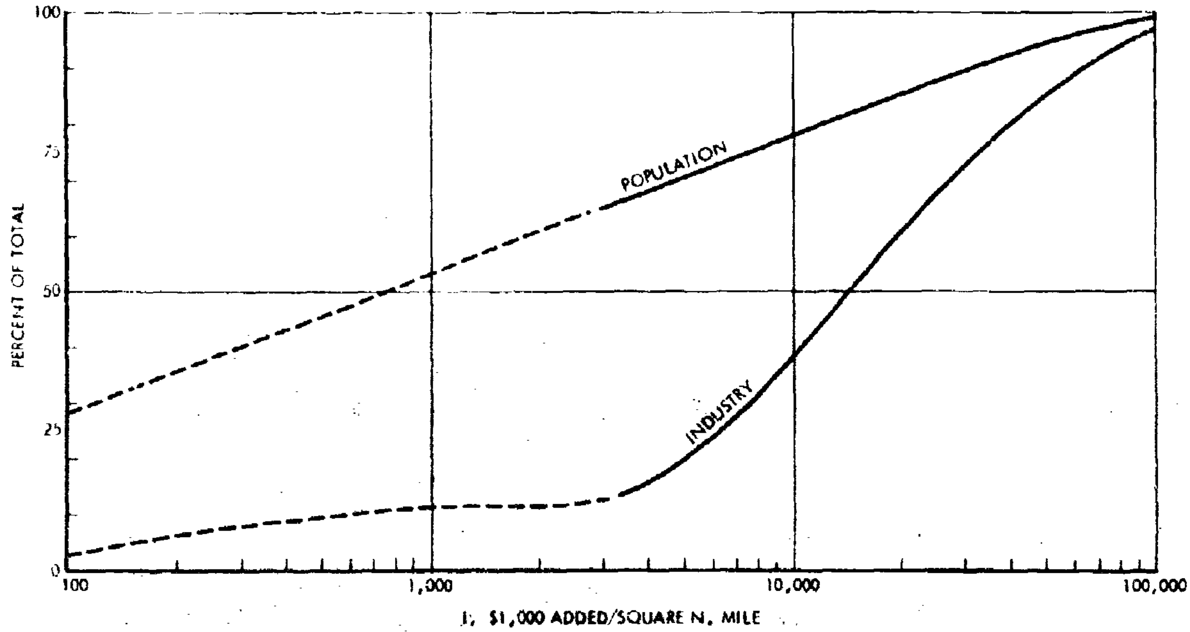

Figure D-8. Percentage of Maximum Population and Industry Located Where the Industrial Density is Less Than I

APPENDIX E

SUGGESTED ADDITIONAL STUDY

APPENDIX E

SUGGESTED ADDITIONAL STUDY

There remain many interesting possibilities for additional exploration of the techniques developed in this study; some of these will be mentioned here.

The first computation requires no program changes, and it uses the capabilities of the program quite completely. This procedure is as follows:

1. Using the parameters from one of the stabilized defenses (with the assumed attack objective set to pure population) or with a set chosen according to the procedure outlined in the stabilization section, compute a nationwide defense and evaluate it. The performance of these defense parameters will now be known for the gross nationwide model.
2. For some city prepare detailed population data, first into cells smaller than two mile squares, and then perhaps into closely spaced "census tract" points.
3. Use the parameters mentioned above as inputs for the defense generation in the targeting model.
4. Use the lambdas corresponding to different nationwide attack levels to generate different attacks on the city. In this way these city attacks can be considered as parts of total nationwide attack. The city evaluation may then be performed for different yields, CEP's, shelter filling modes, times of attack, and attack objectives.

A second area begins with the observation that the effectiveness of shelter deployments seems quite insensitive to the deployment scheme; for example, balanced defense appears to

be a reasonably effective defense. Now that insensitivity has been established, it can be exploited to obtain simple deployment rules as follows:

1. It appears (from time-phasing results) that a defense using 100 psi shelter only would be almost as effective as an unrestricted defense. Several such 100 psi defenses could be generated.

2. The shelter distribution from such an output could be used to generate a curve of either cost allocated per person or average hardness as a function of population density. Since there is only a single shelter type, and since the exact shelter mix deployed makes no difference, such curves completely define the defense.*

3. Fit a simple analytical approximation to the allocation curve. Then, use the approximate rule to generate a shelter deployment, perhaps leaving one or more parameters in the approximation free for an optimization. Such simple rules cannot be found from regular optimization runs, because the optimization procedure inherently generates an "irregular" deployment, choosing one deployment over another to gain even the slightest improvement.

4. The curves found in 2. above themselves would be interesting for comparison between low lambda, medium lambda, high lambda, and stabilized defenses.

Third, there remain areas within the model which could be modified or made more realistic. The exponential evaluation function used in the cell model could be improved on

*For the defenses generated in this study, which consist of shelters of different hardnesses, such curves do not define a defense and hence are not as meaningful.

(perhaps following one of the suggestions made in Appendix B); a different shelter filling mode could be added; or the cost of shelters could be varied as a function of population density. However, the study seems to have eliminated the need to consider certain aspects, such as further industry-like national resources.

Lastly, from consideration of the results of the stabilization procedure, it can be seen that of the three types of stabilization, attack size stabilization and attack objective stabilization achieve their goals satisfactorily. The important factor in a stabilized defense which degraded performance most was attack level stabilization, since the stabilized defenses appear not to offer very much over a range of attack levels compared to a defense tuned to a "middle" value of that range.

In this study the defenses postulated consider blast shelters only; it is at least as interesting to include active defenses also. On the basis of past experience, it is believed that a defense considering active defenses and blast shelters can be made to stabilize the entire attack level range effectively. In such a posture the two types of defenses are complementary: the active defense providing protection against a low level of attack and the blast shelters providing defense against large levels of attack.

APPENDIX F

PARAMETERS FOR CURVES

APPENDIX F

PARAMETERS FOR CURVES

Section IV of the main body of this report (Volume I) contains results plotted from data which was in turn generated by computer runs. This Appendix merely cross correlates the computer runs with those figures based on such computations.

The defense number (*XXXXX) is assigned to each defense deployment; the run number refers to a specific printout. The cost and the lambda combination are specified for each defense, and then the population bases will be shown as NIGHT/DAY, indicating the first and second population bases. The assumed attack objective will be indicated as 70/30, meaning 70% weight on population and 30% weight on industry.

Figures 3 - 6: runs 98, 99, and 105.

Figure 7: runs 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, and 60

Figure 8: run number 151, $5 billion, DAY/DAY, pure population objective; run number 113, $10 billion, DAY/DAY, pure population objective; run number 152, $15 billion, DAY/DAY, pure population objective; run number 115, $20 billion, DAY/DAY, pure population objective; run number 118, 136 and 140, $30 billion, DAY/DAY, pure population objective.

Figure 9: run number 103, $10 billion, NIGHT/NIGHT, pure population; run number 87, $15 billion, NIGHT/NIGHT, pure population; run number 116, $20 billion, NIGHT/NIGHT, pure population; run number 119, $30 billion, NIGHT/NIGHT, pure population; run number 99, unsheltered nighttime population, pure population attack.

Figure 10: run 97, ≈56728, $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/DAY, pure population assumed, compared with the $10 billion optimal curve, run 113.

Figure 11: run 82, ≈1236, $5 billion, lambda = (118270 x .7, 11827 x .3), NIGHT/DAY, 50/50 assumed; run 145, ≈60592, $10 billion, lambda = (78850 x .75, 7885 x .25), DAY/DAY, 70/30 assumed; run 142, ≈41596, $15 billion, lambda = (47300 x .7, 550 x .3), DAY/DAY, 75/25 assumed; run 160, ≈62474, $20 billion, lambda =(40000 x .65, 2500 x .35), DAY/DAY, 80/20 assumed.

Figure 12: run 82, ≈1236, $5 billion, lambda = (118270 x .7, 11827 x .3), NIGHT/DAY, 50/50 assumed; run 95, ≈10930, $10 billion, lambda = (78850 x .7, 7885 x .3), NIGHT/DAY, 50/50 assumed; run 92, ≈46136, $15 billion, lambda = (47300 x .6, 5500 x .4), NIGHT/DAY, 50/50 assumed; run 93, ≈87480, $20 billion, lambda = (31540 x .5, 4500 x .5), NIGHT/DAY, 50/50 assumed.

Figure 13: run 106, ≈40642, $10 billion, lambda = 100000, DAY/DAY, pure population assumed; run 145, ≈60592, $10 billion, lambda =(78850 x .75, 7885 x .25), DAY/DAY, 70/30 assumed.

Figure 14: run 149, ≈30353, $20 billion, lambda = 25000, DAY/DAY, 80/20 assumed; run 160, ≈62474, $20 billion, lambda = (40000 x .65, 2500 x .35), DAY/DAY, 80/20 assumed.

Figure 15: run 98, unsheltered population; run 145, ≈60592, $10 billion, lambda = (78850 x .75, 7885 x .25), DAY/DAY, 70/30 assumed; run 113, the 10.0 DAY/DAY bounds.

Figure 16: runs 79 and 80, ≈95458, $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/DAY, 50/50 assumed.

Figures 17-29: runs 79, 80, 90, 91, 97, and 122, ≈56728, ≈95458, and ≈38442, $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/DAY.

Figure 30: run 79, ≈95458, $10 billion, lambda =(78850 x .6, 7885 x .4), NIGHT/DAY, 50/50 assumed; run 109, ≈18480, lambda = (78850 x .6, 7885 x .4), DAY/DAY, 50/50 assumed.

Figure 31: run 108, ≈95458, $10 billion, lambda = (788504.6, 7885 x.4), NIGHT/DAY, 50/50 assumed; run 117, ≈7426, $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/NIGHT, 50/50 assumed; run 103, $10 billion, NIGHT/NIGHT, pure population.

Figure 32: runs 71, 77, 85, and 86; FINAL shelters, NIGHT/DAY, pure population attack.

Figure 33: run number 155, *56762, $5 billion, lambda = (118270 x .7, 11827 x .3), NIGHT/DAY, 50/50 assumed, time-phased with *27218, run 153, *27218, $10 billion, lambda = (78850 x .7, 7885 x .3), NIGHT/DAY, 50/50 assumed, time-phased with *11364, run 148, *11364, $15 billion, (47300 x .6, 550 x .4), NIGHT/DAY, 50/50 assumed, time-phased with *87480. All other curves identical with Figure 12.

Figure 35: The standard case for the next several figures is run 125, #46521, $10 billion, lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed. The minimax lambda selection is to hold down "twisting" between the standard case and variational cases; run 147, #60402, $10 billion STATCOST, lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed.

Figure 36: run 135, 50% alert evaluation of defense #46521, the standard case; run 127, *417, $10 billion optimized with 50% alert, lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed.

Figure 37: run 135, 15% softer evaluation of defense #46521, the standard case; run 146, #5016, $10 billion optimized with 15% softer shelters, lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed.

Figure 38: run 131, *65717, $10 billion, FINAL-3 shelters (3 added shelter mixes), lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed.

Figure 39: run 4 and 7, *51944, both are HOUSTON shelters, $15 billion, lambda = 10000, DAY/DAY, pure population assumed; total population is 90,619,000.

Figure 40: runs 79 and 133, #95458, $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/DAY, 50/50 assumed.

Figure 43: run 161, *14396; run 93, *87480; both are $15 billion, lambda = (31540 x .5, 4500 x .5), NIGHT/DAY, 50/50 assumed.

Figure 44: runs 137, 138, 143, and 144; run 106, #40642, $10 billion, lambda = 100000 DAY/DAY pure population assumed.

Figure 45: run 125, #46521, $10 billion, lambda = minimax (78850, 7885), NIGHT/DAY, 70/30 assumed.

Figure 47: runs 29 and 30, #59995 and #54375, HOUSTON shelters, $15 billion, lambda = (40000 x.65, 2500 x .35), DAY/DAY, population attack only.

Figure 48: runs 79 and 122, #95458 and #38442; both are $10 billion, lambda = (78850 x .6, 7885 x .4), NIGHT/DAY.

Figure 49: run 150, #87454, $20 billion, lambda = (31540 x .5, 4500 x .5), NIGHT/DAY, pure industry assumed; run 124, #692, lambda minimax (400000,4500), NIGHT/DAY, pure population assumed.

Table IV: runs 8, 10, 11, 13, 14, 17, 18, 19; #9476 13575, 77414, 73800, 52789, and 28249; $15 billion, lambda = 40,000, pure population assumed; total population is 90,619,000.

Table V: runs 82, 100, 101, 102; #1236, 41036, and 49068; $5 billion, lambda = (118270 x .7, 11827 x .3) 50/50 assumed; runs 79, 108, 109, 111, and 117; defense numbers 95458, 18480 and 7426; lambda = (78850 x .6, 7885 x .4), 50/50 assumed; runs 93, 123, 128, 129, and 130; defense numbers 87480, 42734, 90382, and 96390; $20 billion, lambda = (31540 x .5, 4500 x .5), 50/50 assumed.

Tables XII and XIII: run 12, #22740, HOUSTON Shelters; run 20, #36505, HOUSTON2; both are $15 billion, lambda = (40,000 x .522, 2500 x .478), DAY/DAY, pure population assumed; total population is 90,619,000.

APPENDIX G

HOUSTON AND APRIL SHELTER DATA

APPENDIX G

HOUSTON AND APRIL SHELTER DATA

Some of the computer runs were made with shelter data other than the FINAL set. These other two sets of data have labels HOUSTON and APRIL; Table G-1 shows the HOUSTON parameters and Table G-2 the APRIL data.

Table G-1. HOUSTON Parameters

DEFENSE CODE HOUSTON

LISTING OF SHELTER TYPES

SHELTER TYPE	COST	SPREAD OF PSI LEVELS		
UNSHEL	0,0	3,6 • 0,33	6,6 • 0,33	13,0 • 0,33
35 PSI	325,0	40,0 • 0,33	50,0 • 0,33	60,0 • 0,33
100 PSI	379,0	117,0 • 0,33	143,0 • 0,33	175,0 • 0,33
300 PSI	771,0	350,0 • 0,33	430,0 • 0,33	520,0 • 0,33

LISTING OF POSSIBLE SHELTER ALLOCATIONS

DEFENSE OPTION	FRACTION OF POPULATION SUPPLIED EACH SHELTER TYPE		
1	ENTIRE POPULATION SUPPLIED SHELTER TYPE UNSHEL		
2	ENTIRE POPULATION SUPPLIED SHELTER TYPE 35 PSI		
3	ENTIRE POPULATION SUPPLIED SHELTER TYPE 100 PSI		
4	ENTIRE POPULATION SUPPLIED SHELTER TYPE 300 PSI		
5	UNSHEL • 0,50	35 PSI • 0,50	
6	35 PSI • 0,50	100 PSI • 0,50	
7	100 PSI • 0,50	300 PSI • 0,50	
8	UNSHEL • 0,50	100 PSI • 0,50	
9	UNSHEL • 0,50	300 PSI • 0,50	
10	UNSHEL • 0,70	100 PSI • 0,30	
11	UNSHEL • 0,20	100 PSI • 0,80	
12	UNSHEL • 0,80	300 PSI • 0,20	
13	35 PSI • 0,10	100 PSI • 0,50	300 PSI • 0,40
14	UNSHEL • 0,40	100 PSI • 0,40	300 PSI • 0,20
15	UNSHEL • 0,20	100 PSI • 0,50	300 PSI • 0,30

OUTPUT FOR RUN TYPE INPUT

LETHAL AREA TABLE

	PSI											
	3,60	6,60	13,00	40,00	50,00	60,00	117,00	143,00	175,00	350,00	430,00	520,00
0,00	30,68	14,36	6,73	2,39	1,80	1,54	0,84	0,71	0,62	0,34	0,31	0,25
2500,00	41,13	17,87	8,33	2,48	1,92	1,50	1,07	0,90	0,76	0,19	0,14	0,08
3000,00	43,02	18,87	8,84	2,57	1,98	1,88	1,13	0,93	0,71	0,07	0,03	0,00
3500,00	44,96	20,16	9,37	2,67	2,10	1,96	0,87	0,53	0,34	0,00	0,00	0,00
4500,00	51,01	21,76	11,04	2,91	2,30	2,00	0,12	0,00	0,00	0,00	0,00	0,00
5500,00	53,12	23,14	12,65	2,12	0,87	0,53	0,00	0,00	0,00	0,00	0,00	0,00
6500,00	59,68	26,03	13,92	0,62	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
8000,00	64,27	29,09	8,50	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
9000,00	65,68	29,72	5,93	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
10000,00	66,63	27,54	2,39	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

REVISED LETHAL AREA TABLE

	PSI											
	3,60	6,60	13,00	40,00	50,00	60,00	117,00	143,00	175,00	350,00	430,00	520,00
0,00	30,68	14,36	6,73	2,39	1,80	1,54	0,84	0,71	0,62	0,34	0,31	0,25
6500,00	59,68	26,03	13,92	0,62	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

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13. ABSTRACT: This study examines methods of determining blast shelter deployments and of assessing their performance for a variety of nuclear attacks. The goal is not to seek a single optimal deployment, which generally requires making arbitrary assumptions on the nature and size of the attack, thus overlooking the attacker's freedom of choice after a blast shelter program has been deployed. Rather, the study seeks "stabilized" deployments which protect population almost as well as an optimal deployment, even though it is not truly optimal for any specified attack.

The study examines the attacker's freedom to vary force level, time of attack, attack objective, height of burst, and targeting. A quite general and flexible computer model BLAST, based on generalized Lagrange multipliers, generates shelter deployments for the U.S. and computes their effectiveness against attacks in which these factors are varied. In BLAST the nation is considered as a collection of cells two nautical miles square, providing a detailed analysis of the offense/defense interaction.

Volume I summarizes the methodology, results, and conclusions. Volume II contains technical appendixes, including the data employed. Volume III describes BLAST in detail for the analyst and programmer.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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